Maharshi Dayanand University
Rohtak

Ordinance, S.O.E. and Syllabus of M.Sc. Mathematics (1st and 2nd Semester) Examination

Session — 2008-2009

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M. A./M. Sc. Mathematics (Final) w.e.f. 2008-09

Paper Code	Title of the Paper	Regular Theory Marks	Course Internal Marks	DDE Course Theory Marks	Time (Hrs.)
MM 501 A	Integration Theory and Functional Analysis	80	20	100	3
MM 502 A	Partial Differential Equations and Mechanics	80	20	100	3
MM 503 A	Complex Analysis	80	20	100	3
MM 504 A	One paper out of either Group A or Group B	80	20	100	3
MM 505 A	One paper out of either Group C or Group D	80	20	100	3
Total Marks (Final)				500	
Total Marks (Previous)				500	
Grand Marks				1000	

Group A

A_1	Advanced Functional Analysis
A_2	Advanced Discrete Mathematics
A_3	Algebraic Coding Theory
A_4	Wavelets
A_5	Sobolev Spaces
A_6	Harmonic Analysis
A_7	Abstract Harmonic Analysis
A_8	Algebraic Topology

Group B

B_1	Mechanics of Solids
B_2	Continuum Mechanics
B_3	Computational Fluid Dynamics
B_4	Difference Equations
B_5	Information Theory

Group C (Pre-requisite : Group A)

C_1	Theory of Linear Operators
C_2	Analytical Number Theory
C_3	Non-Commulative Rings
C_4	Fuzzy sets and Applications
C_5	Bases in Banach Spaces
C_6	Geometry of Numbers

Group D (Pre-requisite : Group B)

D_1	Fluid Dynamics
D_2	Bio-Mechanics
D_3	Intergal Equations and Boundary Value Probelms
D_4	Dynamical Systems
D_5	Mathematics of Finance and Insurance
D_6	Space Dynamics

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Note 1. : The marks if internal assessment of each paper shall be splited as under :

A) Two class tests of 5 marks each. The class tests will be held in the 1st half of December i.e. before winter break and 1st half of March.

Total Marks for tests = 5+5		= 10 Marks
B) Assignments/term paper & presentation	:	5 marks
C) Attendance	:	5 marks
65% but upto 75%	:	1 mark
More than 75% but upto 85%	:	2 marks
More than 85% but upto 90%	:	3 marks
More than 90% but upto 95%	:	4 marks
Above 95%	:	5 marks

Note 2 : In each paper, there will be 4 sections. Each section will contain 2 or 3 questions so that the total no. of question is 10. A candidate is required to atempt 5 questions selecting at least one question from each section.

Note 3 : There shall be separatre question paper for Regular students (UTD/Maintained Institutions/ Affiliated Institutions) and DDE mode students.

Note 4 : As per UGC recommendations, the teaching program shall be supplemented by tutorials and problem solving sessions for each theory paper. For this purpose, Tutorial classes shall be held for each theory paper in groups of 8 students for half-hour per week.

Note 5 : Optional papers can be offered subject to availability of requisite resources/ faculty and more options can be added depending upon the availability of the staff.

M.SC. MATHEMATICS (FINAL) W.E.F. 2008-2009

MM 501A : Integration Theory and Functional Analysis

Max. Marks : 80 (Regular)
Time : 3 Hours
: 100 (DDE)

Note : Question paper will consist of four sections as indicated below. The candidate will be required to attempt 5 questions selecting at least one question from each section.

Section - I (2 Questions)

Integration Theory

Signed measure, Hahn decomposition theorem, Jordan decomposition theorem, Mutually signed measure, Radon – Nikodyn theorem Lebesgue decomposition, Lebesgue - Stieltjes integral, Product measures, Fubini's theorem.

Baire sets, Baire measure, continuous functions with compact support, Regularity of measures on locally compact spaces, Riesz-Markoff theorem.

Section - II (3 Questions)

Functional Analysis

Normed linear spaces, Metric on normed linear spaces, Holder's and Minkowski's inequality, completeness of quotient spaces of normed linear spaces. Completeness of l_p, L^p, R^n, C^n and C[a,b]. Incomplete normed spaces. Translation invariance, Banach spaces, subspace of a Banach space, Completion of a normed space. Finite dimensional normed linear spaces and compactness.

Bounded linear transformation, Equivalent formulation of continuity, Equivalent norms. Strong convergence and weak convergence, spaces of bounded linear transformations, continuous linear functional, conjugate spaces, Hahn-Banach extension theorem (Real and Complex form), Riesz Representation theorem for bounded linear functionals on L^p and C[a,b].

Section - III (2 Questions)

Second conjugate spaces, Reflexive space, Uniform boundedness principle and its consequences, Open mapping theorem and its application projections, Closed Graph theorem, Equivalent norms, weak and strong convergence, their equivalence in finite dimensional spaces.

Weak sequential compactness, solvability of linear equations in Banach spaces.

Compact operator and its relation with continuous operator. Compactness of linear transformation on a finite dimensional space, properties of compact operators, compactness of the limit of the sequence of compact operators, the closed range theorem.

Section - IV (3 Questions)

Hilbert Spaces: Inner product spaces, Hilbert spaces, Schwarz's inequality, Hilbert space as normed linear space, convex sets in Hilbert spaces, Projection theorem.

Orthonormal sets, Bessel's inequality, Parseval's identity, conjugate of a Hilbert space, Riesz representation theorem in Hilbert spaces, Adjoint of an operator on a Hilbert space, Reflexivity of Hilbert space, Self-adjoint operators, Positive and projection operators, Normal and unitary operators, Projections on Hilbert space, Spectral theorem on finite dimensional space.

Books Recommended

1. H.L. Royden, Real Analysis, MacMillan Publishing Co., Inc., New York, 4th Edition, 1993.
2. E. Kreyszig, Introductory Functional Analysis with Applications, John Wiley.
3. S.K. Berberian, Measure and Integration, Chelsea Publishing Company, New York, 1965.

4. G. Bachman and L. Narici, Functional Analysis, Academic Press, 1966.

5. George F. Simmons, Introduction to Topology and Modern Analysis, McGraw-Hill Book Company, 1963.

MM 502A : Partial Differential Equations and Mechanics

Max. Marks : 80 (Regular)
Time : 3 hours
: 100 (DDE)

Note : Question paper will consist of four sections as indicated below.The candidate will be required to attempt 5 questions selecting at least one question from each section.

Section - I (2 Questions)

Solution of three-dimensional Laplace equation by using the method of separation of variables in terms of Cartesian, cylindrical and spherical coordinates. Method of separation of variables to solve three- dimensional wave equation in Cartesian and spherical coordinates. Use of the method of separation of variables to find steady- state temperature in a rectangular plate, in a disk, in a bar with ends at different temperatures, in a semi-infinite bar, in an infinite plate, in a semi-infinite bar, in an infinite cylinder, in a solid sphere (Relevant topics from the books by Sneddon, and O'Neil).

Section - II (3 Questions)

Kinematics of a rigid body rotating about a fixed point, Euler's theorem, general rigid body motion as a screw motion, moving Coordinate system - rectilinear moving frame, rotating frame of reference, rotating earth.

Moments and products of inertia, Angular momentum of a rigid body, principal axes and principal moment of inertia of a rigid body, kinetic energy of a rigid body rotating about a fixed point, Momental ellipsoid and equimomental systems, coplanar mass distributions, general motion of a rigid body.

Two- dimensional rigid body dynamics – problems illustrating the laws of motion and impulsive motion.

(Relevant topics from the book of Chorlton).

Section - III (2 Questions)

D'Alembert's principle : Constraints, holonomic and non-holonomic systems, Degree of freedom and Generalised coordinates, virtual displacement and virtual work, statement of principle of virtual work (PVW), possible velocity and possible acceleration, D' Alembert's principle.

Lagrangian Formulation : Ideal constraints**,** general equation of dynamics for ideal constraints, Lagrange's equations of the first kind, independent coordinates and generalized forces, Lagrange's equations of the second kind, generalized velocities and accelerations. Uniqueness of solution, variation of total energy for conservative fields.

Lagrange's variable and Lagrangian function L(t, q_i, $\dot{q}_i$), Lagrange's equations for potential forces, generalized momenta p_i- , Hamiltonian variable and Hamiltonian function H(t, q_i, p_i), Donkin's theorem, ignorable coordinates.

Section - IV (3 Questions)

Hamilton canonical equations, Routh variables and Routh function R, Routh's equations, Poisson Brackets and their simple properties, Poisson's identity, Jacobi – Poisson theorem.

Hamilton action and Hamilton's principle, Poincare – Carton integral invariant, Whittaker's equations, Jacobi's equations, Lagrangian action and the principle of least action.

Canonical transformation, necessary and sufficient condition for a canonical transformation, univalent Canonical transformation, free canonical transformation, Hamilton-Jacobi equation, Jacobi theorem, method of separation of variables in HJ equation, Lagrange brackets, necessary and sufficient conditions of canonical character of a transformation in terms of Lagrange brackets, Jacobian matrix of a canonical transformation, conditions of canonicity of a transformation in terms of Poison brackets, Invariance of Poisson Brackets under canonical transformations.

Books Recommended

1. F. Gantmacher, Lectures in Analytic Mechanics, MIR Publishers, Moscow, 1975.
2. P.V. Panat, Classical Mechanics, Narosa Publishing House, New Delhi, 2005.
3. N.C. Rana and P.S. Joag, Classical Mechanics, Tata McGraw- Hill, New Delhi, 1991.
4. Louis N. Hand and Janet D. Finch, Analytical Mechanics, CUP, 1998.
5. Sneddon, I.N., Elements of Partial Differential Equations, McGraw Hill, New York.
6. O'Neil, Peter V., Advanced Engineering Mathematics, ITP.
7. F. Chorlton, Textbook of Dynamics, CBS Publishers, New Delhi.
8. H.F. Weinberger, A First Course in Partial Differential Equations, John Wiley & Sons, 1965.
9. K. Sankra Rao, Classical Mechanics, Prentice Hall of India, 2005.
10. M.R. Speigal, Theoretical Mechanics, Schaum Outline Series.

MM 503A : Complex Analysis

Max. Marks : 80 (Regular)
Time : 3 hours
: 100 (DDE)

Note : Question paper will consist of four sections as indicated below. The candidate will be required to attempt 5 questions selecting at least one question from each section.

Section - I (3 Questions)

Analytic functions, Cauchy-Riemann equation in cartesian and polar coordinates. Power series representation of an analytic function.

Complex integration. Cauchy-Goursat theorem. Cauchy's integral formula. Poisson's integral formula. Higher order derivatives. Morera's theorem. Cauchy's inequality. Liouville's theorem. The fundamental theorem of algebra. Taylor's theorem.

Zeros of an analytic function, Laurent's series. Isolated singularities. Maximum modulus principle. Schwarz lemma. Meromorphic functions. The argument principle. Rouche's theorem.

Section - II (3 Questions)

Calculus of residues. Cauchy's residue theorem. Evaluation of integrals. Branches of many valued functions with special reference to arg z, log z and z^a.

Bilinear transformations, their properties and classifications. Definitions and examples of Conformal mappings.

Integral Functions. Factorization of an integral function. Weierstrass' factorisation theorem. Gamma function and its properties. Stirling formula. Riemann Zeta function. Riemann's functional equation.

Space of analytic functions. Hurwitz's theorem. Montel's theorem. Riemann mapping theorem. Runge's theorem. Mittag-Leffler's theorem.

Section - III (2 Questions)

Analytic Continuation. Natural Boundary. Uniqueness of direct analytic continuation. Uniqueness of analytic continuation along a curve. Power series method of analytic continuation. Schwarz Reflection principle. Germ of an analytic function.

Monodromy theorem and its consequences. Harmonic functions on a disk. Poisson kernel. The Dirichlet problem. Harnack's inequality and theorem. Dirichlet's region. Green's function.

Section - IV (2 Questions)

Canonical product. Jensen's formula. Poisson-Jensen formula. Hadamard's three circles theorem. Growth and order of an entire function. An estimate of number of zeros. Exponent of Convergence. Borel's theorem. Hadamard's factorization theorem.

The range of an analytic function. Bloch's theorem. Schottky's theorem. Little Picard theorem. Montel Caratheodory theorem. Great Picard theorem. Univalent functions. Bieberbach's conjecture (Statement only) and the "1/4 theorem".

Books Recommended

1. H.A. Priestly, Introduction to Complex Analysis, Clarendon Press, Oxford, 1990.
2. J.B. Conway, Functions of one Complex variable, Springer-Verlag, International student-Edition, Narosa Publishing House, 1980.
3. E.T. Copson, An Introduction to the Theory of Functions of a Complex Variable, Oxford University Press, London.

4. E.C. Titchmarsh, The Theory of Functions, Oxford University Press, London.
5. L.V. Ahlfors, Complex Analysis, McGraw Hill, 1979.
6. S. Lang, Complex Analysis, Addison Wesley, 1977.
7. Mark J. Ablowitz and A.S. Fokas, Complex Variables : Introduction and Applications, Cambridge University Press, South Asian Edition, 1998.
8. S. Ponnusam, Foundations of Complex Analysis, Narosa Publishing House, 1997.
9. Ruel V. Churchill and James Ward Brown, Complex Variables and Applications, McGraw-Hill Publishing Company.

MM 504A (Option A_1) : Advanced Functional Analysis

Max. Marks : 80 (Regular)
Time : 3 Hours
: 100 (DDE)

Note : Question paper will consist of four sections as indicated below. The candidate will be required to attempt 5 questions selecting at least one question from each section.

Section - I (3 Questions)

Definition and examples of topological vector spaces, Convex, balanced and absorbing sets and their properties. Minkowski's functional, subspace, product space and quotient space of a topological vector space. Locally convex topological vector spaces, Normable and metrizable topological vector spaces, complete topological vector spaces and Frechet spaces.

Section - II (3 Questions)

Linear transformations, linear functionals and their continuity. Finite dimensional topological vector spaces.

Linear varieties and Hyperplanes, Geometric form of Hahn-Banach Theorem, Uniform boundedness principle. Open mapping theorem and closed Graph Theorem for Frechet spaces, Banach-Alaoglu theorem.

Section - III (2 Questions)

Extreme points and Extremal sets, Krein-Milman's Theorem. Duality, Polar, Bipolar Theorem, Baralled and Bornological spaces, Macekey spaces.

Section - IV (2 Questions)

Semi-reflexive and reflexive topological vector spaces, Montel spaces and Schwarz spaces, Quasi-completeness, Inverse limit and inductive limit of locally convex spaces. Distributions.

Books Recommended

1. J.L. Kelley and Isaac Namioka, Linear Topological Spaces, D. Van Nostrand Company, Inc., 1963.
2. You-Chuen Wong, Introductory Theory of Topological Vector Spaces, Marcel Dekker, Inc., 1992.
3. Laurent Schwarz, Functional Analysis, Courant Institute of Mathematical Sciences, New York University, 1964.
4. G. Kothe, Topological Vector Spaces, Vol. I, Springer, New York, 1969.
5. R. Larsen, Functional Analysis, Marcel Dekker, Inc., New York, 1973.
6. Walter Rudin, Functional Analysis, TMH Edition, 1974.
7. H.H. Schaefer, Topological Vector Spaces, MacMillan, New York, 1966, Reprinted, Springer, N.Y., 1971.

MM 504A (Option A_2) Advanced Discrete Mathematics

Max. Marks: 80 (Regular)
Time : 3 Hours
: 100 (DDE)

Note : Question paper will consist of four sections as indicated below. The candidate will be required to attempt 5 questions selecting at least one question from each section.

Section - I (2 Questions)

Formal Logic – Statements. Symbolic Representation and Tautologies. Quantifier, Predicates and Validty. Propositional Logic.

Semigroups & Monoids-Definitions and Examples of Semigroups and Monoids (including those pertaining to concatenation operation). Homomorphism of semigroups and monoids. Congruence relation and Quotient Semigroups. Subsemigroup and submonoids. Direct products. Basic Homomorphism Theorem. Pigeonhole principle, principle of inclusion and exclusion, derangements.

Lattices- Lattices as partially ordered sets. Their properties. Lattices as Algebraic systems. Sublattices, Direct products, and Homomorphisms. Some Special Lattices e.g., Complete. Complemented and Distributive Lattices.

Section - II (2 Questions)

Boolean Algebras – Boolean Algebras as Lattices. Various Boolean Identities. The switching Algebra example. Subalgebras, Direct Products and Homomorphisms. Join-irreducible elements. Atoms and Minterms. Boolean Forms and Their Equivalence. Minterm Boolean Forms, Sum of Products Canonical Forms. Minimization of Boolean Functions. Applications of Boolean Algebra to Switching Theory (using AND, OR & NOT gates). The Karnaugh Map method.

Section - III (3 Questions)

Graph Theory – Definiton of (undirected) graphs, paths, circuits, cycles and subgraphs, induced subgraphs, degree of a vertex, isomorphism, walks, paths and circuits, connectivity, bipartite and complete bipartite graphs, Kurtowski's theorem (statement only) and its use, Euler Graphs, Euler's theorem on the existence of Eulerian paths and circuits, Hamiltonian graph, matrix representation of graphs, directed graphs, indegree and outdegree of a vertex, weighted undirected graphs, Dijkstra's algorithm, strong connectivity and Warshall's algorithm. Trees, spanning trees, cut sets, fundamental cut sets, and cycles. Minimal spanning trees and Kruskal's algorithm, directed trees, search trees, tree traversals. Planar graphs and their properties, Euler formula for connected planar graphs.

Section - IV (3 Questions)

Introductory Computability Theory - Finite state machines and their transition table diagrams, equivalence of finite state machines, reduced machines, homomorphism, finite automata acceptors, non-deterministic finite automata and equivalence of its power to that of deterministic finite automata Moore and Mealy machines.

Grammars and Languages – Phrase-structure grammar rewriting rules, derivations, sentential forms, language generated by a grammar, regular, context-free and context sensitive grammars and languages, regular sets, regular expressions and pumping lemma, Kleene's theorem.

Books Recommended

1. Babu Ram, Discrete Mathematics, Vinayak Publishers and Distributors, Delhi, 2004.
2. J.P. Tremblay & R. Manohar, Discrete Mathematical Structures with Applications to Computer Science, McGraw-Hill Book Co., 1997.
3. J.L. Gersting, Mathematical Structures for Computer Science, (3rd edition), Computer Science Press, New York.

4. Seymour Lipschutz, Finite Mathematics (International edition 1983), McGraw-Hill Book Company, New York.

5. C.L. Liu, Elements of Discrete Mathematics, McGraw-Hilll Book Co.

MM 504A (Option A_3) : Algebraic Coding Theory

Max. Marks: 80 (Regular)
Time : 3 hours
: 100 (DDE)

Note : Question paper will consist of four sections as indicated below. The candidate will be required to attempt 5 questions selecting at least one question from each section.

Section - I (3 Questions)

The communication channel. The Coding Problem. Types of Codes. Block Codes. Error-Detecting and Error-Correcting Codes. Linear Codes. Hamming Metric. Description of Linear Block Codes by Matrices. Dual Codes. Hamming Codes, Golay Codes, perfect and quasi-perfect codes. Modular Representation. Error-Correction Capabilities of Linear Codes. Bounds on Minimum Distance for Block Codes. Plotkin Bound. Hamming Sphere Packing Bound. Varshamov-Gilbert – Sacks Bound. Bounds for Burst-Error Detecting and Correcting Codes.

Section - II (2 Questions)

Tree Codes. Convolutional Codes. Description of Linear Tree and Convolutional Codes by Matrices. Standard Array. Bounds on minimum distance for Convolutional Codes. V.G.S. bound. Bounds for Burst-Error Detecting and Correcting Convolutional Codes. The Lee metric, packing bound for Hamming code w.r.t. Lee metric.

Section - III (3 Questions)

Cyclic Codes. Cyclic Codes as ideals. Matrix

Description of Cyclic Codes. Hamming and Golay Codes as Cyclic Codes. Error Detection with Cyclic Codes. Error-Correction procedure for Short Cyclic Codes. Short-ended Cyclic Codes. Pseudo Cyclic Codes. Quadratic residue codes of prime length, Hadamard Matrices and non-linear Codes derived from them. Product codes. Concatenated codes.

Section - IV (2 Questions)

Code Symmetry. Invariance of Codes under transitive group of permutations. Bose-Chaudhary-Hoquenghem (BCH) Codes. BCH bounds. Reed-Solomon (RS) Codes. Majority-Logic Decodable Codes. Majority- Logic Decoding. Singleton bound. The Griesmer bound. Maximum – Distance Separable (MDS) Codes. Generator and Parity-check matrices of MDS Codes. Weight Distribution of MDS code. Necessary and Sufficient conditions for a linear code to be an MDS Code. MDS Codes from RS codes. Abramson Codes. Closed-loop burst-error correcting codes (Fire codes). Error Locating Codes.

Books Recommended

1. Ryamond Hill, A First Course in Coding Theory, Oxford University Press, 1986.
2. Man Young Rhee, Error Correcting Coding Theory, McGraw Hill Inc., 1989.
3. W.W. Petersona nd E.J. Weldon, Jr., Error-Correcting Codes. M.I.T. Press, Cambridge Massachuetts, 1972.
4. E.R. Berlekamp, Algebraic Coding Theory, McGraw Hill Inc., 1968.
5. F.J. Macwilliams and N.J.A. Sloane, Theory of Error Correcting Codes, North-Holand Publishing Company.
6. J.H. Van Lint, Introduction to Coding Theory, Graduate Texts in Mathematics, 86, Springer, 1998.
7. L.R. Vermani, Elements of Algebraic Coding Theory, Chapman and Hall, 1996.

MM 504A (Option A_4) : Wavelets

Max. Marks: 80 (Regular)
Time : 3 hours
: 100 (DDE)

Note : Question paper will consist of four sections as indicated below. The candidate will be required to attempt 5 questions selecting at least one question from each section.

Section - I (2 Questions)

Different ways of constructing wavelets, orthonormal bases generated by a single function ; the Balian-Low theorem. Smooth projections in $L^2(R)$. Local sine and cosine bases and the construction of some wavelets. The unitary folding operators and the smooth projections. (Chapter 1 of Hernandez and Weiss [1])

Section - II (2 Questions)

Multiresolution analysis and construction of wavelets. Construction of compactly supported wavelets and estimates for its smoothness. Orthonormality. Completeness. Characterization of Lemarie-Meyer wavelets (Chapter 2 and sections 3.1, 3.2 and 3.3 of Chapter 3 of Hernandez and Weiss [1]).

Section - III (3 Questions)

Franklin wavelets and Spline wavelets on the real line. Orthonormal bases of piecewise linear continuous functions for $L^2(T)$. Orthonormal bases of periodic splines. Periodization of wavelets defined on the real line. (Chapter 4 of Hernandez and Weiss [1]).

Characterizations in the theory of wavelets – The basic equations and some of its applications. Characterizations of MRA wavelets, low-pass filters and scaling function. Non-existence of smooth wavelets in $H^2(R)$. (Chapter 7 of Hernandez and Weiss [1]).

Section - IV (3 Questions)

Frames – The reconstruction formula and the Balian-Low theorem for frames. Frames from translations and dilations. Smooth frames for H^2R. (Chapter 8 of Hernandez and Weiss [1]).

Discrete transforms and algorithms – The discrete and the fast Fourier transforms. The discrete and the fast cosine transforms. The discrete version of the local sine and cosine bases. Decomposition of reconstruction algorithms for wavelets (Chapter 9 of Hernandez and Weiss [1]).

Books Recommended

1. Eugenio Hernandez and Guido Weiss, A first Course on Wavelets, CRC Press, New York, 1996.
2. C.K. Chui, An Introduction to Wavelets, Academic Press, 1992.
3. I. Daubechies, Ten Lectures on Wavelets, CBS-NSF Regional Conferences in Applied Mathematics, 61, SIAM, 1992.
4. Y. Meyer, Wavelets, Algorithms and Applications (translated by R.D. Rayan, SIAM, 1993).

MM 504A (Option A_5) : Sobolev Spaces

Max. Marks : 80 (Regular)
Time : 3 hours
: 100 (DDE)

Note : Question paper will consist of four sections as indicated below. The candidate will be required to attempt 5 questions selecting at least one question from each section.

Section - I (2 Questions)

Distributions – Test function spaces and distributions, convergence distributional derivatives.

Fourier Transform – L^1-Fourier transform. Fourier transform of a Gaussian, L^2-Fourier transform, Inversion formula. L^p-Fourier transform, Convolutions.

Section - II (3 Questions)

Sobolev Spaces - The spaces $W^{l,p}_{¥}$ (W) and $W^{l,p}$ (W). Their simple characteristic properties, density results. Min and Max of $W^{l,p}$ – functions. The space H^1(W) and its properties, density results.

Imbedding Theorems - Continuous and compact imbeddings of Sobolev spaces into Lebesgue spaces. Sobolev Imbedding Theorem, Rellich – Kondrasov Theorem.

Other Sobolev Spaces - Dual Spaces, Fractional Order Sobolev spaces, Trace spaces and trace theory.

Section - III (3 Questions)

Weight Functions - Definiton, motivation, examples of practical importance. Special weights of power type. General Weights.

Weighted Spaces - Weighted Lebesgue space P(W, s) , and their properties.

Section - IV (2 Questions)

Domains - Methods of local coordinates, the classes C^o, $C^{o,k}$, Holder’s condition, Partition of unity, the class K (x_0) including Coneproperty.

Inequalities – Hardy inequality, Jensen’s inequality, Young’s inequality, Hardy-Littlewood - Sobolev inequality, Sobolev inequality and its various versions.

Books Recommended

1. R.A. Adams, Sobolev Spaces, Academic Press, Inc. 1975.
2. S. Kesavan, Topics in Functional Analysis and Applications, Wiley Eastern Limited, 1989.

3. A. Kufner, O. John and S. Fucik, Function Spaces, Noordhoff International Publishing, Leyden, 1977.
4. A. Kufner, Weighted Sobolev Spaces, John Wiley & Sons Ltd., 1985.
5. E.H. Lieb and M. Loss, Analysis, Narosa Publishing House, 1997.
6. R.S. Pathak, A Course in Distribution Theory and Applications, Narosa Publishing House, 2001.

MM 504A (Option A_6) : Harmonic Analysis

Max. Marks : 80 (Regular)
Time : 3 hours
: 100 (DDE)

Note : Question paper will consist of four sections as indicated below. The candidate will be required to attempt 5 questions selecting at least one question from each section.

Section - I (3 Questions)

Basic properties of topological groups, subgroups, quotient groups and connected groups. Discussion of Haar Measure without proof on R, T, Z, and some simple matrix groups. $L^1(G)$ and convolution with special emphasis on $L^1(R)$, $L^1(T)$, $L^1(Z)$. Approximate identities. Fourier series. Fejer's theorem. The classical kernels. Fejer's Poisson's and Dirichlet's summability in norm and pointwise summability. Fatou's Theorem.

Section - II (2 Questions)

The inequalities of Hausdorff and Young. Examples of conjugate function series. The Fourier transform. Kernels on R. The Plancherel theorem on R. Plancherel measure on R, T, Z. Maximal ideal space of $L^1(R)$, $L^1(T)$ and $L^1(Z)$.

Section - III (3 Questions)

Hardy spaces on the unit circle. Invariant subspaces. Factoring. Proof of the F. and M. Riesz theorem. Theorems of Beurings and Szego in multiplication operator form. Structure of inner and outer functions. The inequalities of hardy and Hilbert. Conjugate functions. Theorems of Kolmogorov & Zygmund and M. Riesz and Zygmund on conjugate functions. The conjugate function as a singular integral.

Section - IV (2 Questions)

Statement of the Burkholder-Gundy Silverstein Theorem on T. Maximal functions of Hardy and Littlewood Translation. The Theorems of Wiener and Beurling. The Titchmarsh Convolution Theorem. Wiener-s Tauberian Theorem. Spectral sets of bounded functions.

Books Recommended

1. Henry Helson, Harmonic Analysis, Addison-Wesley 1983, Second Edition, Hindustan Pub. Corp., 1994.
2. E. Hewitt and K.A. Ross, Abstract Harmonic Analysis Vol. 1, 4^{th} Edition, Springer-Verlag, 1993.
3. Y. Katznelson, An Introduction to Harmonic Analysis, John Wiley, 1968.
4. P. Koosis, Introduction of H^p Spaces, Cambridge University Press.

MM 504A (Option A_7) : Abstract Harmonic Analysis

Max. Marks : 80 (Regular)

Time : 3 hours

: 100 (DDE)

Note : Question paper will consist of four sections as indicated below. The candidate will be required to attempt 5 questions selecting at least one question from each section.

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(To be framed later on)

MM 504A (Option A_8) : Algebraic Topology

Max. Marks : 80 (Regular)

Time : 3 hours

: 100 (DDE)

Note : Question paper will consist of four sections as indicated below. The candidate will be required to attempt 5 questions selecting at least one question from each section.

Section - I (3 Questions)

Fundamental group function, homotopy of maps between topological spaces, homotopy equivalence, contractible and simple connected spaces, fundamental groups of S^1, and $S^1 \times S^1$ etc.

Calculation of fundamental group of S^n, $n > 1$ using Van Kampen's theorem, fundamental groups of a topological group. Brouwer's fixed point theorem, fundamental theorem of algebra, vector fields on planer sets. Frobenius theorem for 3 x 3 matrices.

Covering spaces, unique path lifting theorem, covering homotopy theorems, group of covering transformations, criterian of lifting of maps in terms of fundamental groups, universal covering, its existence, special cases of manifolds and topological groups.

Singular homology, reduced homology, Eilenberg Steenrod axioms of homology (no proof for homotopy invariance axiom, excision axiom and exact sequence axiom) and their application, relation between fundamental group and first homology.

Section - II (3 Questions)

Calculation of homology of S^n, Brouwer's fixed point theorem for $f : E^n ® E^n$, application spheres, vector fields, Mayer-Vietoris sequence (without proof) and its applications.

Mayer Vietoris sequence (with proof) and its application to calculation of homology of graphs, torus and compact surface of genus g, collared pairs, construction of spaces by attaching of cells, spherical complexes with examples of S^n, r-leaved rose, torus, **RP^n, CP^n** etc.

Computation of homology of **RP^n, CP^n**, torus, suspension space, XVY, compact surface of genus g and non-orientable surface of genus h using Mayer Vietoris sequence, Betti numbers and Euler characteristics and their calculation for S^n, r-leaved rose, **RP^n, CP^n, S^2 x S^2, X + Y** etc.

Section - III (2 Questions)

Singular cohomology modules, Kronecker product, connecting homomorphism, contra-functoriality of singular cohomology modules, naturality of connecting homomorphism, exact cohomology sequence of pair, homotopy invariance, excision properties, cohomology of a point. Mayer Vietoris sequence and its application in computation of cohomology of **S^n, RP^n, CP^n, torus, compact surface of genus g and** non-orientable compact surface.

Section - IV (2 Questions)

Compact connected 2-manifolds, their orientability and non-orientability, examples, connect sum, construction of projective space and Klein's bottle from a square, Klein's bottle as unity of two Mobius strips, canonical form of sphere, torus and projective palnes. Kelin's bottle. Mobious strip, triangulation of compact surfaces.

Classification theorem for compact surfaces, connected sum of torus and projective plan as the connected sum of three projective planes. Euler characteristic as a topological invariation of compact surfaces. Connected sum formula, 2-manifolds with boundary and their classification Euler characteristic of a bordered surface, models of compact bordered surfaces in **R^3 .**

Books Recommended

1. James R. Munkres, Topology – A First Course, Prentice Hall of India Pvt. Ltd., New Delhi, 1978.
2. Marwin J. Greenberg and J.R. Harper, Algebraic Topology – A First Course, Addison-Wesley Publishing Co., 1981.
3. W.S. Massey, Algebraic Topology – An Introduction, Harcourt, Brace and World Inc. 1967, SV, 1977.

MM 504A (Option B_1) : Mechanics of Solids

Max. Marks : 80 (Regular)
Time : 3 hours
: 100 (DDE)

Note : Question paper will consist of four sections as indicated below. The candidate will be required to attempt 5 questions selecting at least one question from each section.

Section - I (2 Questions)

Analysis of Stress : Stress vector, stress components. Cauchy equations of equilibrium. Stress tensor. Symmetry of stress tensor. Stress quadric of Cauchy. Principal stress and invariants. Maximum normal and shear stresses. Mohr's diagram.

Analysis of Strain : Affine transformations. Infinitesimal affine deformation. Geometrical interpretation of the components of strain. Strain quadric of Cauchy. Principal strains and invariants. General infinitesimal deformation. Saint-Venant's equations of Compatibility. Finite deformations. Examples of uniform dilatation, simple extension and shearing strain.

Section - II (3 Questions)

Equations of Elasticity : Generalized Hooke's law. Hooke's law in media with one plane of symmetry, orthotroic and transversely isotropic media, Homogeneous isotropic media. Elastic moduli for isotropic media. Equilibrium and dynamic equations for an isotropic elastic solid. Beltrami-Michell compatibility equations. Strain energy function. Clapeyron's theorem, Reciprocal theorem of Betti and Rayleigh. Theorems of minimum potential energy. Theorems of minimum complementary energy.

Extension and bending of beams : Extension of beams by longitudinal forces, beam stretched by its own weight, bending of beams by terminal couples, bending of a beam by a transverse load at the centroid of the end section along a principal axis.

Section - III (3 Questions)

Two-dimensional Problems : Plane strain and Plane stress. Generalized plane stress. Airy stress function for plane strain problems. General solutions of a Biharmonic equation using fourier transform as well as in terms of two analytic functions. Stresses and displacements in terms of complex potentials. Problems of semi-infinite solids with displacements or stresses prescribed on the plane boundary. Thick walled tube under external and internal pressures. Rotating shaft.

Torsional Problems : Torsion of cylindrical bars. Torsional rigidity. Torsion and stress functions. Lines of shearing stress. Torsion of a bar of arbitrary cross-section, Simple problems related to circle, ellipse and equilateral triangle cross-section. Circular groove in a circular shaft. **Variational Methods** : Deflection of elastic string, central line of a beam and elastic membrane. Variational problem related to biharmonic equation. Solution of Euler's equation : Ritz, Galerkin and Kantorovich methods.

Section - IV (2 Questions)

Elastic Waves : Simple harmonic progressive waves, scalar wave equation, progressive type solutions, plane waves and spherical waves, stationary type solutions in Cartesian and cylindrical coordinates. Propagation of waves in an unbounded isotropic elastic solid. P, SV and SH waves. Wave propagation in two-dimensions. Elastic surface waves such as Rayleigh and Love waves. Radial vibrations of a homogeneous solid elastic circular cylinder and cylindrical shell. Reflection of a P-wave at a free boundary. Reflection and refraction of a P-wave at a solid-solid boundary.

Books Recommended

1. I.S. Sokolnikoff, Mathematical Theory of Elasticity, Tata McGraw Hill Publishing Company Ltd., New Delhi, 1977.
2. Teodar M. Atanackovic and Ardeshiv Guran Theory of Elasticity for Scientists and Engineers Birkhausev, Boston, 2000.
3. Y.C. Fung, Foundations of Solid Mechanics, Prentical Hall, New, Delhi, 1965.
4. C.A. Coulson, Waves.
5. Jeffreys, H. , Cartesian tensor.
6. M.R. Speigel, Vector Anlaysis, Schaum Outline Series.

Paper 504A (Option B_2) : Continuum Mechanics

Max. Marks : 80 (Regular)
Time : 3 hours
: 100 (DDE)

Note : Question paper will consist of four sections as indicated below. The candidate will be required to attempt 5 questions selecting at least one question from each section.

Section - I (3 Questions)

Analysis of Stress : Stress vector, stress components, Cauchy equations of equilibrium, stress tensor, symmetry of stress tensor, principal stress and invariants.

Analysis of Strain : Affine transformation, infinitesimal affine deformation, geometric interpretation of the components of strain, principal strains and invariants, general infinitesimal deformation, equations of compatibility, finite deformations.

Equations of Elasticity : Generalized Hooke's law, Hooke's law in media with one plane of symmetry, orthotropic and transversely isotropic media, Homogeneous isotropic media, elastic moduli for isotropic media, equilibrium and dynamical equations for an isotropic elastic solid, Beltrami – Michell compatibility equations.

Section - II (3 Questions)

Two-Dimensional Elasticity : Plane strain, plane stress, Airy stress function, problem of half-plane loaded by uniformly distributed load, problem of thick wall tube under the action of internal and external pressures.

Thermoelasticity : Stress-strain relation for thermo-elasticity, Navier equations for thermoelasticity, thermal stresses in a long circular cylinder and in a sphere.

Viscoealsticity : Viscoelastic models – Maxwell model, Kelvin model and Standard linear solid model. Creep compliance and relaxation modulus, Hereditary integrals, visco-elastic stress-strain relations, correspondence principle and its application to the deformation of a viscoelastic thick-walled tube in plane strain.

Section - III (2 Questions)

Fluid Dynamics : Viscous stress tensor, Stokesian and Newtonian fluid, Basic equation of viscous flow, Navier stokes equations, specialized fluid, steady flow, irrotational flow, potential flow, Bernoullies equation, Kelvin's theorem (As Chapter 7 of the Book by Mase and Mase).

Section - IV (2 Questions)

Plasticity : Basic concepts, yield criteria, yield surface, equivalent stress and equivalent strain, elastic – plastic stress-strain relation, plastic stress- strain relation, plastic flow of anisotropic material, special cases of plane stress, plane strain and axisymmetry.

Books Recommended

1. S. Valliappan, Continuum Mechanics, Fundamentals, Oxford & IBH Publishing Company, 1981.
2. G.T. Mase and G.E. Mase, Continuum Mechanics for Engineers, CRC Press, 1999.
3. Atanackovic, T.M. A. Guran, Theory of Elasticity for scientists and Engineers, Birkhausev, 2000.
4. D.S. Chandrasekharaiah, Continuum Mechanics, Academic Press, Prism Books Pvt. Ltd., Bangalore.
5. L.S. Srinath, Advanced Mechanics of Fields, Tata McGraw-Hill, New Delhi.

MM 504A (Option B_3) : Computational Fluid Dynamics

Max. Marks : 80 (Regular)
Time : 3 hours
: 100 (DDE)

Note : Question paper will consist of four sections as indicated below. The candidate will be required to attempt 5 questions selecting at least one question from each section.

Section - I (2 Questions)

Basic equations of Fluid dynamics. Analytic aspects of partial differential equations- classification, boundary conditions, maximum principles, boundary layer theory.

Finite difference and Finite volume discretizations. Vertex-centred discretization. Cell-centred discretization. Upwind discretization. Nonuniform grids in one dimension.

Section - II (3 Questions)

Finite volume discretization of the stationary convection-diffusion equation in one dimension. Schemes of positive types. Defect correction. Non-stationary convection-diffusion equation. Stability definitions. The discrete maximum principle.

Incompressible Navier-Stokes equations. Boundary conditions. Spatial discretization on collocated and on staggered grids. Temporal discretization on staggered grid and on collocated grid.

Section - III (3 Questions)

Iterative methods. Stationary methods. Krylov subspace methods. Multigrade methods. Fast Poisson solvers. Iterative methods for incompressible Navier-Stokes equations.

Shallow-water equations – One and two dimensional cases. Godunov 's order barrier theorem. Linear schemes. Scalar conservation laws.

Section - IV (2 Questions)

Euler equation in one space dimension – analytic aspects. Approximate Riemann solver of Roe. Osher scheme. Flux splitting scheme. Numerical stability. Jameson – Schmidt – Turkel scheme. Higher order schemes.

Books Recommended

1. P. Wesseling : Principles of Computational Fluid Dynamics, Springer Verlag, 2000.
2. J.F. Wendt, J.D. Anderson, G. Degrez and E. Dick, Computational Fluid Dynamics : An Introduction, Springer-Verlag, 1996.
3. J.D. Anderson, Computational Fluid Dynamics : The basics with applications, McGraw-Hill, 1995.

MM 504A (Option B_4) : Difference Equations

Max. Marks : 80 (Regular)
Time : 3 hours
: 100 (DDE)

Note : Question paper will consist of four sections as indicated below. The candidate will be required to attempt 5 questions selecting at least one question from each section.

Section - I (2 Questions)

Introduction, Difference Calculus – The difference operator, Summation, Generating functions and approximate summation.

Linear Difference Equations - First order equations. General results for linear equations. Equations with constant coefficients. Applications. Equations with variable coefficients.

Section - II (2 Questions)

Stability Theory - Initial value problems for linear systems. Stability of linear systems. Stability of nonlinear systems. Chaotic behaviour.

Asymptotic methods - Introduction, Asymptotic analysis of sums. Linear equations. Nonlinear equations.

Section - III (3 Questions)

Self-adjoint second order linear equations –Introduction. Sturmian Theory. Green's functions. Disconjugacy. The Riccati Equations. Oscillation.

Sturm-Liouville problems - Introduction, Finite Fourier Analysis. A non- homogeneous problem.

Discrete Calculus of Variations - Introduction. Necessary conditions. Sufficient Conditions and Disconjugacy.

Section - IV (3 Questions)

Nonlinear equations that can be linearized. The z-transform.

Boundary value problems for Nonlinear equations. Introduction. The Lipschitz case. Existence of solutions. Boundary value problems for differential equations.

Partial Differential Equations.

Discretization of Partial Differential Equations.

Solution of Partial Differential Equations.

Books Recommended

1. Walter G. Kelley and Allan C. Peterson- Difference Equations. An Introduction with Applications, Academic Press Inc., Harcourt Brace Joranovich Publishers, 1991.

2. Calvin Ahlbrandt and Allan C. Peterson. Discrete Hamiltonian Systems, Difference Equations, Continued Fractions and Riccatti Equations. Kluwer, Boston, 1996.

MM 504A (Option B_5) : Information Theory

Max. Marks : 80 (Regular)
Time : 3 hours
: 100 (DDE)

Note : Question paper will consist of four sections as indicated below. The candidate will be required to attempt 5 questions selecting at least one question from each section.

Section - I (2 Questions)

Measure of Information – Axioms for a measure of uncertainty. The Shannon entropy and its properties. Joint and conditional entropies. Transformation and its properties.

Noiseless coding - Ingredients of noiseless coding problem. Uniquely decipherable codes. Necessary and sufficient condition for the existence of instantaneous codes. Construction of optimal codes.

Section - II (2 Questions)

Discrete Memoryless Channel - Classification of channels. Information processed by a channel. Calculation of channel capacity. Decoding schemes. The ideal observer. The fundamental theorem of Information Theory and its strong and weak converses.

Continuous Channels - The time-discrete Gaussian channel. Uncertainty of an absolutely continuous random variable. The converse to the coding theorem for time-discrete Gaussian channel. The time-continuous Gaussian channel. Band-limited channels.

Section - III (3 Questions)

Some imuitive properties of a measure of entropy – Symmetry, normalization, expansibility, boundedness, recursivity, maximality, stability, additivity, subadditivity, nonnegativity, continuity, branching, etc. and interconnections among them. Axiomatic characterization of the Shannon entropy due to Shannon and Fadeev.

Information functions, the fundamental equation of information, information functions continuous at the origin, nonnegative bounded information functions, measurable information functions and entropy. Axiomatic characterizations of the Shannon entropy due to Tverberg and Leo. The general solution of the fundamental equation of information. Derivations and their role in the study of information functions.

Section - IV (3 Questions)

The branching property. Some characterisations of the Shannon entropy based upon the branching property. Entropies with the sum property. The Shannon inequality. Subadditivie, additive entropies.

The Renji entropies. Entropies and mean values. Average entropies and their equality, optimal coding and the

Renji entropies. Characterisation of some measures of average code length.

Books Recommended

1. R. Ash, Information Theory, Interscience Publishers, New York, 1965.
2. F.M. Reza, An Introduction to Information Theory, MacGraw-Hill Book Company Inc., 1961.
3. J. Aczela dn Z. Daroczy, On Measures of Information and their Characterizations, Academic Press, New York.

MM 505A (Option C_1) : Theory of Linear Operators

Max. Marks : 80 (Regular)
Time : 3 hours
: 100 (DDE)

Note : Question paper will consist of four sections as indicated below. The candidate will be required to attempt 5 questions selecting at least one question from each section.

Section - I (2 Questions)

Spectral theory in normed linear spaces, resolvent set and spectrum, spectral properties of bounded linear operators, Properties of resolvent and spectrum, Spectral mapping theorem for polynomials, Spectral radius of a bounded linear operator on a complex Banach space. Elementary theory of Banach algebras. Properties of Banach algebras. (Relevant portions of Chapter 7 of Kreyszig).

Section - II (2 Questions)

General properties of compact linear operators. Spectral properties of compact linear operators on normed spaces. Behaviour of compact linear operators with respect to solvability of operator equations. Fredholm type theorems. Fredholm alternative theorem. Fredholm alternative for integral equations. (Relevant oprtions of Chapter 8 of Kreyszig).

Section - III (3 Questions)

Spectral properties of bounded self-adjoint linear operators on a complex Hilbert space, Positive operators, Monotone sequence theorem for bounded self-adjoint operators on a complex Hilbert space. Square roots of a positive operator. Projection operators, Spectral family of a bounded self-adjoint linear operator and its properties. Spectral representation of bounded self adjoint linear operators. Spectral theorem. Properties of the spectral family of a bounded self-adjoint linear operator. (The relevant portion of Chapter 9 of Kreyszig).

Section - IV (3 Questions)

Unbounded linear operators in Hilbert Space. Hellinger - Toeplitz theorem. Hilbert adjoint operators. Symmetric and self-adjoint linear operators, Closed linear operators and closures. Spectrum of an unbounded self-adjoint linear operator. Spectral theorem for unitary and self-adjoint linear operators. Multiplication operator and differentiation operator (Relevant portion of Chapter 10 of Kreyszig).

Spectral measures. Spectral integrals. Regular spectral measures. Real and complex spectral measures. Complex spectral integrals. Description of the spectral subspaces. Characterization of spectral subspaces. The spectral theorem for bounded normal operators (Relevant portion of sections 36-44 of Chapter 2 of Halmos).

Books Recommended

1. E. Kreyszig, Introductory Functional Analysis with Applications, John-Wiley & Sons, New York, 1978.
2. P.R. Halmos, Introduction to Hilbert Space and the Theory of Spectral Multiplicity, Second-Edition, Chelsea Publishing Co., New York, 1957.
3. N. Dunford and J.T. Schwartz, Linear Operators -3 Parts, Interscience/Wiley, New York, 1958-71.
4. G. Bachman and L. Narici, Functional Analysis, Academic Press, York, 1966.

MM 505A (Option C_2) : Analytical Number Theory

Max. Marks : 80 (Regular)
Time : 3 hours
: 100 (DDE)

Note : Question paper will consist of four sections as indicated below. The candidate will be required to attempt 5 questions selecting at least one question from each section.

Section - I (3 Questions)

Distribution of primes. Fermat's and Mersenne numbers, Farey series and some results concerning Farey series. Approximation of irrational numbers by rations, Hurwitz's theorem. Irrationality of e and p. Diophantine equations ax + by = c. $x^2+y^2 = z^2$ and $x^4+y^4 = z^4$. The representation of number by two or four squares. Warig's problem, Four square theorem, the numbers g(k) & G(k). Lower bounds for g(k) & G(k). Simultaneous linear and non-linear congruences Chinese Remainder Theorem and its extension. (Relevant portions from the Books Recommended at Sr. No. 1 and 4)

Section - II (2 Questions)

Quadratic residues and non-residues. Legender's Symbol. Gauss Lemma and its applications. Quadratic Law of Reciprocity Jacobi's Symbol. The arithmetic in Z_n. The group U_n . Congruences with prime power modulus, primitive roots and their existence. The group $U_p{}^n$ (p-odd) and $U_2{}^n$. The group of quadratic residues Q_n , quadratic residues for prime power moduli and arbitrary moduli. The algebraic structure of U_n and Q_n- (Scope as in Chapters 4, 6 and 7 of Recommended Book at Sr. No. 5)

Section - III (3 Questions)

Riemann Zeta Function z(s) and its convergence. Application to prime numbers. z (s) as Euler's product.

Evaluation of z (2) and z (2k). Dirichlet series with simple properties. Eulers products and Dirichlet products, Introduction to modular forms (Scope as in Chapters 8 and 9 of Recommended Book at Sr. No.5).

Algebraic number and Integers : Gaussian integers and its properties. Primes and fundamental theorem in the ring of Gaussian integers. Integers and fundamental theorem in Q (w) where w^3 = 1. Algebraic fields. Primitive polynomials. The general quadratic field Q(Öm), Units of Q(Ö2). Fields in which fundamental theorem is false. Real and complex Euclidean fields. Fermat'' theorem i the ring of Gaussian integers. Primes of Q(Ö2) and Q(Ö5) Series of Fibonacci and Lucas. Luca'' test for the primality of the mersenne primes. (Relevant sections of Recommended Book at Sr. No. 1).

Section - IV (2 Questions)

Arithmetic functions f(n) t(n), s(n) and s_k(n), u(n), N(n), I(n). Definition and examples and simple properties. Perfect numbers the Mobius inversion formula. The Mobius function m_n, The order and average order of the function f(n), t(n) and s(n). The functions L(n), y(n) and J(n) Bertrand Postulate, Merten's theorem, Selberg's theorem and Prime number Theorem (Scope as in Chapter 8 of Recommended Book at Sr. No. 5 and Recommended books at Sr. No. 1 and 4).

Books Recommended

1. Hardy, G.H. and Wright, E.M., An Introduction to the Theory of Numbers
2. Burton, D.M., Elementary Number Theory.
3. McCoy, N.H., The Theory of Number by McMillan.
4. Niven, I. And Zuckermann, H.S., An Introduction to the Theory of Numbers.
5. Gareth, A. Jones and J. Mary Jones, Elementary Number Theory, Springer Ed. 1998.

MM 505A (Option C_3) : Non-Commutative Rings

Max. Marks : 80 (Regular)
Time : 3 hours
: 100 (DDE)

Note : Question paper will consist of four sections as indicated below. The candidate will be required to attempt 5 questions selecting at least one question from each section.

(Detailed syllabi to be framed later on)

MM 505A (Option C_4) : Fuzzy Sets and their Applications

Max. Marks : 80 (Regular)
Time : 3 hours
: 100 (DDE)

Note : Question paper will consist of four sections as indicated below. The candidate will be required to attempt 5 questions selecting at least one question from each section.

Section - I (2 Questions)

Fuzzy sets - Basic definitions, a-level sets. Convex fuzzy sets. Union and Intersection of Fuzzy sets. Types of fuzzy sets, Cartesian product, Bounded sum and difference, t-norms and t-conorms. Fuzzy measures, measures of fuzziness. Chapters 2, 3 & 4 of Zimmermann).

Section - II (3 Questions)

Zadeh's extension Principle, Type 2 fuzzy sets, Fuzzy numbers, Fuzzy relations on set, composition of fuzzy relations, minimum-maximum composition, Fuzzy graphs, Similarity relation. Fuzzy functions on Fuzzy sets, Extrema of Fuzzy function, Fuzzy integration, Fuzzy differentiation, Possibility Measure, Necessity measure, Probability of Fuzzy event, Possibility theory versus Probability theory (Chapters 5, 6, 7, 8 of Zimmermann).

Section - III (3 Questions)

Fuzzy logic - An overview of classical logic, Linguistic variables and hedges. Approximate reasoning - An overview of Fuzzy expert system. Fuzzy languages. Uncertainty modeling in expert systems, Applications of Fuzzy set theory in expert systems (Chapter 9 and Sections 10.1 of Chapter 10 of Zimmermann).

Section - IV (2 Questions)

An Introduction to Fuzzy control – Fuzzy controllers. Process of Fuzzy control, Application of Fuzzy control. Pattern recognition. Fuzzy decision making, Fuzzy linear programming. Fuzzy dynamical programming (Section 10.2 of Chapter 10, Chapter 11 and Section 12.1-12.3 of Chapter 12 of Zimmermann).

Books Recommended

1. H.J. Zimmerman, Fuzzy Set Theory and its Applications, Allied Publishers Ltd., New Delhi, 1991.
2. John Yen, Reza Langari, Fuzzy Logic - Intelligence, Control and Information, Pearson Education.

MM 505A (Option C_5) : Bases in Banach Spaces

Max. Marks : 80 (Regular)
Time : 3 hours
: 100 (DDE)

Note : Question paper will consist of four sections as indicated below. The candidate will be required to attempt 5 questions selecting at least one question from each section.

Section - I (3 Questions)

Hamel bases. The coefficient functionals associated to a basis. Schauder bases. Bounded bases and normalized bases. Examples of bases in concrete Banach spaces.

Biorthogonal systems. Associated sequences of partial sum operators -E-complete, regular and irregular biorthogonal systems. Characterizations of regular biorthogonal systems. Basic sequences. Banach space (separable or not) and basic sequence.

Some types of linear independence of sequences - Linearly independent (finitely) W-linearly independent and minimal sequences of elements in Banach spaces. Their relationship together with examples and counter-examples.

Problem of uniqueness of basis - Equivalent bases, Stability theorems of Paley-Winer type. Block basic sequences with respect to a sequence (basis) and their existence. Bessaga-Pelczynski theorem.

Section - II (2 Questions)

Properties of strong duality. Weak bases and weak Schauder bases in a Banach space. Weak basis theorem. Weak* bases in conjugate spaces and their properties.

Shrinking bases and boundedly complete bases together with their relationship.

Different types of convergence in a Banach space. Unconditional bases. Unconditional basis sequences. Symmetric Bases.

Bases and structure of the space Bases and completeness. Bases and reflexivity.

Section - III (3 Questions)

Generalized bases. Generalized basic sequences. Boundedly complete generalized bases and shrinking generalized bases. M-Bases (Markusevic bases) and M-Basic sequences.

Bases of subspaces (Decomposition-Existence of a decomposition in Banach spaces. The sequences of co-ordinate projections associated to a decomposition. Schauder decompositions. Characterization of a Schauder decomposition. Example that a decomposition is not always Schauder. Various possibilities for a Banach space to possess a Schauder decomposition. Shrinking decompositions, boundedly complete decompositions and unconditional decompositions. Reflexivity of Banach spaces having a Schauder decomposition).

Bases and decompositions in the space C[0,1].

Section - IV (2 Questions)

Best approximation in Banach spaces. Existence and uniqueness of element of best approximation by a subspace. Proximinal subspaces. Semi Cebysev subspaces and Cebysev subspaces.

Monotone and strictly monotone bases, co-monotone and strictly co-monotone bases.

Examples and counter-examples.

T-norm, K-norm and KT-norm on Banach spaces having bases. Their characterization in terms of monotone and comonotone bases. Various equivalent norms on a Banach space in terms of their bases.

Books recommended

1. Jurg t. Marti, Introduction to Theory of Bases, Springer Tracts in Natural Philosophy 18, 1969.
2. Ivan Singer, Bases in Banach Spaces I, Springer-Verlag, Berlin, Vol. 154 1970.
3. Ivan Singer, Bases in Banach Spaces II, Springer-Verlag, Berlin, 1981.
4. J. Linderstrauss and I. Tzafriri, Classical banach Spaces (Sequence spaces), Springer Verlag, Berlin, 1977.
5. Ivan Singer, Best Approximation in Normed Linear Spaces by Elements of Linear Spaces, Springer-Verlag, Berlin, 1970.

MM 505A (Option C_6) : Geometry of Numbers

Max. Marks : 80 (Regular)
Time : 3 hours
: 100 (DDE)

Note : Question paper will consist of four sections as indicated below. The candidate will be required to attempt 5 questions selecting at least one question from each section.

(Detailed syllabi to be framed later on)

MM 505A (Option D_1) : Fluid Dynamics

Max. Marks : 80 (Regular)
Time : 3 hours
: 100 (DDE)

Note : Question paper will consist of four sections as indicated below. The candidate will be required to attempt 5 questions selecting at least one question from each section.

Section - I (3 Questions)

Kinematics - Eulerian and Lagrangian methods. Stream lines, path lines and streak lines. Velocity potential. Irrotational and rotational motions. Vortex lines. Equation of continuity. Boundary surfaces.

Acceleration at a point of a fluid. Components of acceleration in cylindrical and spherical polar co-ordiantes, Pressure at a point of a moving fluid. Euler's and Lagrange's equations of motion. Bernoulli's equation. Impulsive motion. Stream function.

Acyclic and cyclic irrotation motions. Kinetic energy of irrotational flow. Kelvin's minimum energy theorem. Axially symmetric flows. Liquid streaming part a fixed sphere. Motion of a sphere through a liquid at rest at infinity. Equation of motion of a sphere.

Section - II (3 Questions)

Three-dimensional sources, sinks, doublets and their images. Stoke's stream function.

Irrotational motion in two-dimensions. Complex velocity potential. Milne-Thomson circle theorem. Two-dimensional sources, sinks, doublets and their images. Blasius theorem. Two- dimensional irrotation motion produced by motion of circular and co-axial cylinders in an infinite mass of liquid.

Vortex motion. Kelvin's circulation theorem. Vorticity equation. Motions due to circular and rectilinear vortices. Vortex doublet. Image of a vortex. Single and double infinite rows of vortices.

Section - III (2 Questions)

Stress components in a real fluid. Relations between rectangular components of stress. Gradients of velocity. Connection between stresses and gradients of velocity. Navier-Stoke's equations of motion. Equations of motion in cylindrical and spherical polar co-ordinates.

Plane Poiseuille and Couette flows between two parallel plates. Theory of lubrication. Flow through tubes of uniform cross-section in form of circle, annulus, ellipse and equilateral triangle under constant pressure gradient. Unsteady flow over a flat plate.

Section - IV (2 Questions)

Dynamical similarity. Inspection analysis. Reynolds number. Dimensional analysis. Buckingham p-theorem. Prandtl's boundary layer. Boundary layer equation in two-dimensions. Blasius solution. Boundary layer thickness, displacement thickness, momentum thickness. Karman integral conditions. Karman-Pohlmansen method. Separation of boundary layer flow.

Books Recommended

1. W.H. Besaint and A.S. Ramasey, A Treatise on Hydromechanics, Part II, CBS Publishers, Delhi, 1988.
2. F. Chorlton, Text Book of Fluid Dynamics, C.B.S. Publishers, Delhi, 1985
3. O'Neill, M.E. and Chorlton, F., Ideal and Incompressible Fluid Dynamics, Ellis Horwood Limited, 1986.
4. O'Neill, M.E. and Chorlton, F., Viscous and Compressible Fluid Dynamics, Ellis Horwood Limited, 1989.
5. S.W. Yuan, Foundations of Fluid Mechanics, Prentice Hall of India Private Limited, New Delhi, 1976.
6. H. Schlichting, Boundary-Layer Theory, McGraw Hill Book Company, New York, 1979.
7. R.K. Rathy, An Introduction to Fluid Dynamics, Oxford and IBH Publishing Company, New Delhi, 1976.

MM 505A (Option D_2) : Biomechanics

Max. Marks : 80 (Regular)
Time : 3 hours
: 100 (DDE)

Note : Question paper will consist of four sections as indicated below. The candidate will be required to attempt 5 questions selecting at least one question from each section.

Section - I (2 Questions)

Newton's equations of motion. Mathematical modeling. Continuum approach. Segmental movement and vibrations. Lagrange's equations. Normal modes of vibration. Decoupling of equations of motion.

Flow around an airfoil. Flow around bluff bodies. Steady state aeroelastic problems. Transient fluid dynamics forces due to unsteady motion. Flutter. Kutta-Joukowski theorem. Circulation and vorticity in the wake. Vortex system associated with a finite wing in nonsteady motion. Thin wing in steady flow.

Section - II (2 Questions)

Blood flow in heart, lungs, arteries, and veins. Field equations and boundary conditions. Pulsatile flow in Arteries. Progressive waves superposed on a steady flow. Reflection and transmission of waves at junctions. Velocity profile of a steady flow in a tube. Steady laminar flow in an elastic tube. Velocity profile of Pulsatile flow. The Reynolds number, Stokes number, and Womersley number. Systematic blood pressure. Flow in collapsible tubes.

Section - III (3 Questions)

Micro-and macrocirculation Rheological properties of blood. Pulmonary capillary blood flow. Respiratory gas flow. Intraction between convection and diffusion. Dynamics of the ventilation system.

Laws of thermodynamics. Gibbs and Gibbs – Duhem equations. Chemical potential. Entropy in a system with heat and mass transfer. Diffusion, filtration, and fluid movement in interstitial space in thermodynamic view. Diffusion from molecular point of view.

Section - IV (3 Questions)

Mass transport in capillaries, tissues, interstitial space, lymphatics, indicator dilution method, and peristalsis. Tracer motion in a model of pulmonary microcirculation.

Descrption of internal deformation and forces. Equations of motion in Lagrangian description. Work and strain energy. Calculation of stresses from strain energy function.

Stress, strain and stability of organs. Stress and strains in blood vessels. Strength, Trauma, and tolerance.

Shock loading and structural response. Vibration and the amplification spectrum of dynamic structural response.

Books Recommended

Y.C. Fung, Biomechanics, Springer-Verlag, New York Inc., 1990.

MM 505A (Option D_3) : Integral Equations and Boundary Value Problems

Max. Marks : 80 (Regular)
Time : 3 hours
: 100 (DDE)

Note : Question paper will consist of four sections as indicated below. The candidate will be required to attempt 5 questions selecting at least one question from each section.

Section - I (3 Questions)

Definition of integral equations and their classification. Eigenvalues and eigenfunctions. Convolution integral. Fredholm integral equations of the second kind with separable kernels and their reduction to a system of algebraic equations. Fredholm alternative. Fredholm theorem, Fredholm alternative theorem. An approximate method.

Method of successive approximations. Iterative scheme for Fredholm integral equations of the second kind. Neumann series, iterated kernels, resolvent kernel, Iterative scheme for Volterra integral equations of the second kind. Conditions of uniform convergence and uniqueness of series solution.

Classical Fredholm theory. Fredholm's first, second and third theorems.

(Relevant topics from the chapters 1 to 4 of the book by R.P. Kanwal).

Section - II (3 Questions)

Applications to ordinary differential equations. Initial value problems transformed to volterra integral equations. Boundary value problems equivalent to Fredholm integral equations. Dirac delta function.

Construction of Green's function for a BVP associated with a nonhomogeneous ordinary differential equation of second order with homogeneous boundary conditions by using the method of variation of parameters, Basic four properties of the Green's function. Alternative procedure for construction of a Green's function by using its basic four properties. Green's function approach for IVP for second order equations. Green's function for higher order differential equations. Modified Green's function.

Application to Partial Differential Equations. Integral representation formulas for the solutions of the Laplace and Poisson equations. Newtonian single layer and double layer potentials. Interior and exterior Dirichlet and Neumann problems for Laplace equation. Green's function for Laplace equation in a free space as well as in a space bounded by a grounded vessel. Integral equation formulation of BVPs for Lapalce equation. The Helmholftz equation.

(Relevant topics from the chapters 5 and 6 of the book by R.P. Kanwal).

Section - III (2 Questions)

Symmetric kernels. Complex Hilbert space. Orthonormal system of functions. Rierz-Fischer theorem (statement only). Fundamental properties of eigenvalues and eigenfunctions for symmetric kernels. Expansion in eigenfunctions and bilinear form. A necessary and sufficient condition for a symmetric L_2-kernel to be separable. Hilbert-Schmidt theorem. Definite and indefinite kernels. Mercer's theorem (statement only). Solution of integral equations with symmetric kernels by using Hilbert-Schmidt theorem.

Singular integral equations. The Abel integral equation. Inversion formula for singular integral equation with kernel of the type $[h(s) – h(t)]^{-a}$ with $0 < a < 1$. Cauchy principal value for integrals. Solution of the Cauchy type singular integral equations. The Hilbert kernel. Solution of the Hilbert-type singular integral equations.

(Relevant topics from the Chapters 7 and 8 of the book by R.P. Kanwal).

Section - IV (2 Questions)

Integral transform methods. Fourier transform. Laplace transform. Applications to Volterra integral equations with convolution type kernels. Hilbert transforms and their use to solve integral equations.

Applications to mixed BVP's. Two-part BVP's, Three-part BVP's Generalized two-part BVP's.

Perturbation method. Its applications to Stokes and Oseen flows, and to Navier-Cauchy equations of elasticity for elastostatic and elastodynamic problems.

(Relevant topics from the chapters 9 to 11 of the book by R.P. Kanwal).

Books Recommended

1. Kanwal, R.P., Linear Integral Equations – Theory and Technique, Academic Press, 1971.
2. Kress, R., Linear Integral Equations, Springer-Verlag, New York, 1989.
3. Jain, D.L. and Kanwal, R.P., Mixed Boundary Value Problems in Mathematical Physics.
4. Smirnov, V.I., Integral Equations and Partial Differential Equations, Addison-Wesley, 1964.
5. Jerri, A.J., Introduction to Integral Equations with Applications, Second Edition, John-Wiley & Sons, 1999.
6. Kanwal, R.P., Linear Integration Equations, (2nd Ed.) Birkhauser, Boston, 1997.

MM 505A (Option D_4) : Dynamical Systems

Max. Marks : 80 (Regular)

Time : 3 hours

: 100 (DDE)

Note : Question paper will consist of four sections as indicated below. The candidate will be required to attempt 5 questions selecting at least one question from each section.

Section - I (2 Questions)

Orbit of a map, fixed point, equilibrium point, periodic point, configuration space and phase space.

Section - II (2 Questions)

Origin of bifurcation, stability of a fixed point, equilibrium point, concept of limit cycle and torus, Hyperbolicity, Quadratic map, Feigenbaum's universal constant.

Section - III (3 Questions)

Turning point, transcritical, pitch work, Hopf bifurcations, period doubling phenomena.

Nonlinear oscillators - conservative system. Hamiltonian system. Various types of oscillators in nonlinear system. Solutions of nonlinear differential equations.

Section - IV (3 Questions)

Phenomena of losing stability, Quasiperiodic motion, Topological study of nonlinear differential equations, Poincare map.

Randomness of orbits of a dynamic system, chaos, strange attractors, various roots to chaos. Onset mechanism of turbulence.

Books Recommended

1. Rober L. Dawaney, An Introduction to Chaotic Dynamical Systems, Addison Wesley Publishing Co Inc., 1989.

2. P.G. Drazin, Nonlinear Systems, Cambridge University Press, 1993.
3. V.I. Arnold, Dynamical Systems V. Bifurcation Theory and Catastrophy Theory, Springer-Verlag, 1992.
4. V.I. Arnold, Dynamical Systems III – Mathematical Aspects of Classical and Celestial Mechanics, 2nd Edition, Springer-Verlag, 1992.
5. D.K. Arrowosmith, Introduction to Dynamical Systems, Cambridge

University Press, 1990.

MM 505A (Option D_5) : Mathematics for Finance and Insurance

Max. Marks : 80 (Regular)
Time : 3 hours
: 100 (DDE)

Note : Question paper will consist of four sections as indicated below. The candidate will be required to attempt 5 questions selecting at least one question from each section.

Section - I (3 Questions)

Application of Mathematics in Finance

Financial Management – AN overview. Nature and Scope of Financial Management. Goals of Financial Management and main decisions of financial management. Difference between risk, speculation and gambling.

Time value of Money - Interest rate and discount rate. Present value and future value- discrete case as well as continuous compounding case. Annuities and its kinds.

Meaning of return. Return as Internal Rate of Return (IRR). Numerical Methods like Newton Raphson Method to calculate IRR. Measurement of returns under uncertainty situations. Meaning of risk. Difference between risk and uncertainty. Types of risks. Measurements of risk.

Calculation of security and Portfolio Risk and Return-Markowitz Model. Sharpe's Single Index Model- Systematic Risk and Unsystematic Risk.

Taylor series and Bond Valuation. Calculation of Duration and Convexity of bonds.

Section - II (3 Questions)

Application of Mathematics in Insurance

Insurance Fundamentals – Insurance defined. Meaning of loss. Chances of loss, peril, hazard, and proximate cause in insurance. Costs and benefits of insurance to the society and branches of insurance-life insurance and various types of general insurance. Insurable loss exposures- feature of a loss that is ideal for insurance.

Life Insurance Mathematics – Construction of Morality Tables. Computation of Premium of Life Insurance for a fixed duration and for the whole life.

Determination of claims for General Insurance – Using Poisson Distribution and Negative Binomial Distribution –the Polya Case.

Determination of the amount of Claims of General Insurance – Compound Aggregate claim model and its properties, and claims of reinsurance. Calculation of a compound claim density function. F-recursive and approximate formulae for F.

Section - III (2 Questions)

Financial Derivatives – Futures. Forward. Swaps and Options. Call and Put Option. Call and Put Parity Theorem. Pricing of contingent claims through Arbitrage and Arbitrage Theorem.

Financial Derivatives – An Introduction; Types of Financial Derivatives- Forwards and Futures; Options and its kinds; and SWAPS.

The Arbitrage Theorem and Introduction to Portfolio Selection and Capital Market Theory : Static and Continuous – Time Model.

Pricing Arbitrage - A Single-Period option Pricing Model ; Multi-Period Pricing Model - Cox – Ross - Rubinstein Model; Bounds on Option Prices.

The Ito's Lemma and the Ito's integral.

Section - IV (2 Questions)

The Dynamics of Derivative Prices - Stochastic Differential Equations (SDEs) - Major Models of SDEs : Linear Constant Coefficient SDEs; Geometric SDEs; Square Root Process; Mean Reverting Process and Omstein - Uhlenbeck Process.

Martingale Measures and Risk - Neutral Probabilities : Pricing of Binomial Options with equivalent martingale measures.

The Black-Scholes Option Pricing Model - using no arbitrage approach, limiting case of Binomial Option Pricing and Risk-Neutral probabilities.

The American Option Pricing - Extended Trading Strategies; Analysis of American Put Options; early exercise premium and relation to free boundary problems.

Books Recommended

1. Aswath Damodaran, Corporate Finance - Theory and Practice, John Wiley & Sons, Inc.
2. John C. Hull, Options, Futures, and Other Derivatives, Prentice-Hall of Indian Private Limited.
3. Sheldon M. Ross, An Introduction to Mathematical Finance, Cambridge University Press.
4. Mark S. Dorfman, Introduction to Risk Management and Insurance, Prentice Hall, Englwood Cliffs, New Jersey.

5. C.D. Daykin, T. Pentikainen and M. Pesonen, Practical Risk Theory for Actuaries, Chapman & Hall.
6. Salih N. Neftci, An Introduction to the Mathematics of Financial Derivatives, Academic Press, Inc.
7. Robert J. Elliott and P. Ekkehard Kopp, Mathematics of Financial Markets, Sprigner-Verlag, New York Inc.
8. Robert C. Merton, Continuous – Time Finance, Basil Blackwell Inc.
9. Tomasz Rolski, Hanspter Schmidli, Volker Schmidt and Jozef Teugels, Stochastic Processes for Insurance and Finance, John Wiley & Sons Limited.

MM 505A (Option D_6) : Space Dynamics

Max. Marks : 80 (Regular)
Time : 3 hours
: 100 (DDE)

Note : Question paper will consist of four sections as indicated below. The candidate will be required to attempt 5 questions selecting at least one question from each section.

Section - I (2 Questions)

Basic Formulae of a spherical triangle - The two-body Problem : The Motion of the Center of Mass. The relative motion. Kepler's equation. Solution by Hamilton Jacobi theory.

The Determination of Orbits – Laplace's Gauss Methods.

Section - II (2 Questions)

The Three-Body problem – General Three Body Problem. Restricted Three Body Problem. Jacobi integral. Curves of Zero velocity. Stationary solutions and their stability.

Section - III (3 Questions)

The n-Body Problem – The motion of the centre of Mass. Classical integrals.

Perturbation – Osculating orbit, Perturbing forces, Secular & Periodic perturbations. Lagrange's Planetory Equations in terms of pertaining forces and in terms of a perturbed Hamiltonian.

Motion of the moon – The perturbing forces. Perturbations of Keplerian elements of the Moon by the Sun.

Section - IV (3 Questions)

Flight Mechanics – Rocket Performance in a Vacuum. Vertically ascending paths. Gravity Twin trajectories. Multi stage rocket in a Vacuum. Definiton spertinent to single stage rocket. Performance limitations of single stage rockets, Definitions pertinent to multi stage rockets. Analysis of multi stage rockets neglecting gravity. Analysis of multi stage rockets including gravity.

Rocket Performance with Aerodynamic forces.

Short range non-lifting missiles. Ascent of a sounding rocket. Some approximate performance of rocket-powered air-craft.

Books Recommended

1. J.M. A. Danby, Fundamentals of Celestial Mechanics. The MacMillan Company, 1962.
2. E. Finlay, Freundlich, Celestial Mechanics. The MacMillan Company, 1958.
3. Theodore E. Sterne, An Introduction of Celestial Mechanics, Intersciences Publishers. INC., 1960.
4. Arigelo Miele, Flight Mechanics – Vol . 1 - Theory of Flight Paths, Addison-Wesley Publishing Company Inc., 1962.