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MPHDURS-EE-2013

SUBJECT : Mathematics

18/9/13

A

Sr. No. 10081

Time : **1¼ Hours** Max. Marks : **100** Total Questions : **100**

Candidate's Name __________ Date of Birth __________

Father's Name __________ Mother's Name __________

Roll No. (in figures) __________ (in words) __________

Date of Examination __________

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1. The function $D: R \to R$ such that

$$D(x) = \begin{cases} 1 \text{ if } x \in Q \\ 0 \text{ if } x \notin Q \end{cases}$$

is known as :

(1) Step Function
(2) Simple Function
(3) Characteristic Function
(4) Dirichlet's Function

2. Every convergent sequence of measurable functions is nearly uniformly convergent. This result is known as :

(1) 1st principle of measurability
(2) Littlewood's 2nd principle of measurability
(3) Littlewood's third principle
(4) Egorov's theorem

3. If a_n and b_n are sequences of extended real numbers and $a_n \leq b_n$ for all n sufficiently large. Which of the following is ***not true*** ?

(1) $\lim \inf a_n \geq \lim \inf b_n$
(2) $\lim \inf a_n \leq \lim \inf b_n$
(3) $\lim \sup a_n \leq \lim \sup b_n$
(4) None of these

4. The composition of two Lebesgue measurable functions is :

(1) not necessarily Lebesgue measurable
(2) Borel measurable
(3) always measurable
(4) always Lebesgue measurable

5. Every uniformly continuous function is :

(1) Absolutely continuous
(2) Not absolutely continuous
(3) Not Continuous
(4) None of these

6. Which of the following statements is ***not correct*** ?

(1) Ch. function of irrational numbers in [0, 1] is Riemann integrable
(2) Ch. functions are simple functions
(3) Ch. function of the set E of rational numbers in [0, 1] is measurable
(4) None of the above

7. Let A be the set of algebraic numbers. Then the outer measure of A is equal to :

(1) ∞
(2) a finite measure
(3) zero
(4) outer measure of the set of real numbers

8. The axiom of choice was formulated in 1904 by :

(1) Riemann (2) Ernst Zermelo (3) G. H. Moore (4) George Cantor

9. The result "Let (–1, 1) be interval of convergence for the power series $\sum a_n x^n$. If $\sum_{n=0}^{\infty} a_n = S$, then $\lim_{x \to 1-0} \sum_{n=0}^{\infty} a_n x^n = S$" is known as :

(1) Uniqueness theorem (2) Weierstrass's theorem

(3) Tauber's theorem (4) Abel's theorem

10. If a function f is convex and $f(0) \leq 0$, then :

(1) f is superadditive on the positive half axis

(2) f is additive

(3) f is subadditive on the positive half axis

(4) f is superconvex

11. The result "Let $< f_n >$ be a sequence of non-negative measurable functions which converge almost everywhere on a set E to a function f. Then $\int_E f \leq \underline{\lim} \int_E f_n$" is known as :

(1) F. Riesz Theorem

(2) Bounded Convergence Theorem

(3) Fatou's Lemma

(4) Lebesgue Monotone Convergence Theorem

12. The members of the smallest σ-algebra which contains all of the open sets are called :

(1) Lebesgue sets (2) Borel sets

(3) σ-open sets (4) Lebesgue measurable sets

13. For $o \leq p \leq 1$, the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^p}$ is :

(1) convergent but not absolutely (2) convergent

(3) absolutely convergent (4) oscillatory

14. The series $\sum_{n=1}^{\infty} \frac{\cos n\theta}{n^p}$ converges uniformly for all values of θ if :

(1) $p \geq 1$ (2) $p < 1$ (3) $p \leq 1$ (4) $p > 1$

15. Outermeasure is a set function whose domain is :

(1) P(R) (2) R

(3) Collection of all measurable sets (4) Collection of all continuous functions

16. Which of the following is ***not true*** ?

(1) Every absolutely continuous function is of bounded variation

(2) Every bounded function is of bounded variation

(3) Every monotone function on $[a, b]$ is of bounded variation

(4) Every function of bounded variation is bounded

17. The word 'Topologi' was introduced in Germany in 1847 by :

(1) George Cantor (2) Johann Benedict

(3) Kazimierz Kuratowski (4) Felix Hausdorff

18. A function which is analytic for all finite values of Z and bounded is :

(1) a constant (2) zero (3) a function of Z (4) continuous

19. The result "The order of a canonical product is equal to the exponent of convergence of its zeros" is known as :

(1) Borel's theorem (2) Jensen's formula

(3) Bloch's theorem (4) Morera's theorem

20. The constant

$$r = \lim_{n\to\infty}\left(1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} - \log n\right)$$

is called :

(1) Euler's constant (2) Euler's number

(3) Lebesgue constant (4) Lebesgue number

21. If a function $f(z)$ is analytic except at finite number of singularities (including that at infinity), then the sum of residues of these singularities is :

(1) $2\pi i$ (2) πi (3) finite (4) zero

22. The transformation $f(z) = x - iy$ is :

(1) analytic (2) conformal (3) isogonal (4) none of these

23. The set of all bilinear transformation under the product of transformations form a :

(1) Monoid (2) Abelion group (3) Semi group (4) Non-Abelion group

24. The function $f(z)=e^{1/z}$ has essential singularity at :

(1) $Z=1$ (2) $Z=0$ (3) $Z=2$ (4) $Z=-1$

25. Which of the following statement is ***not true*** ?

(1) Exponential function is analytic

(2) Absolute value function when defined on the set of real or complex numbers is analytic

(3) Power functions are analytic

(4) Any polynomial is an analytic function

26. The simple poles of Gamma function are at :

(1) $Z=0, 1, 2,\dots, n, \dots$ (2) $Z=0, -1, -2,\dots, -n, \dots$

(3) $Z=1, 2,\dots, n, \dots$ (4) None of these

27. If $f(z)$ and $g(z)$ are analytic inside and on a closed contonic and $|g(z)|<|f(z)|$ on C, then $f(z)$ and $f(z)+g(z)$ have same :

(1) value (2) number of poles

(3) number of singularities (4) number of zeros

28. The residue of $f(z)=\dfrac{z^3}{z^2-1}$ at $z=\infty$ is :

(1) -1 (2) 1 (3) 0 (4) 3

29. The Taylor series of the function $\log(1+Z)$ about the point $Z=0$ is convergent for the region :

(1) $|Z|\le 1$ (2) $|Z|<1$ (3) $|Z|\ge 1$ (4) $|Z|>1$

30. Which of the following statement is ***not correct*** ?

(1) v is a harmonic conjugate of u if and only if u is a harmonic conjugate of $-v$.

(2) An analytic function with constant modulus is constant.

(3) If v is a harmonic conjugate of u in the same domain, then u is a harmonic conjugate of v.

(4) Both the real and imaginary parts of an analytic function are harmonic.

31. The product of two odd permutations is :

(1) even and odd (2) odd (3) even (4) none of these

32. A group has almost one composition series. This result is known as :

(1) Cayley's theorem (2) Sylow's theorem

(3) Lagrange's theorem (4) Jordan-Holder theorem

33. If every non-constant polynomial over a field F has all its roots in F, then F is :

(1) Algebraically Closed Field (2) Prime Field

(3) Perfect Field (4) None of the above

34. Let $R = F[x]$ be a polynomial ring over a field F. Then R is :

(1) Artinian but not Noetherian (2) Artinian and Noetherian both

(3) Neither Artinian nor Noetherian (4) Noetherian but not Artinian

35. Which of the following is a prime field ?

(1) Q (2) R (3) $\mathbb{C}$ (4) Z_n

36. Let G be a commutative group having composition series. Then G must be :

(1) Infinite (2) Finite

(3) Finite with $G' = G$ (4) Infinite with $Z(G) = \langle e \rangle$

37. Let M be a simple R-module and $T \in \text{Home}_R(M, M)$ such that $T \neq 0$, then :

(1) $I_m(T) = O$ (2) $\ker(T) = M$ (3) T is singular (4) T is non-singular

38. A composition series for a group is :

(1) Central series (2) Derived series

(3) Solvable series (4) None of these

39. The degree of the splitting field of the polynomial $f(x) = x^{10} - 1$ over Q is :

(1) 10 (2) 4 (3) 6 (4) 8

40. Any group of order 15 is :

(1) Abelian (2) Simple (3) Cyclic (4) p-group

41. The basis and the degree of the extension $Q(\sqrt{2}, \sqrt{3})$ over Q is :

(1) $\{\sqrt{2}, \sqrt{3}\}$, 4 (2) $\{1, \sqrt{2}, \sqrt{3}\}$, 4

(3) $\{1, \sqrt{2}, \sqrt{3}, \sqrt{6}\}$, 4 (4) $\{1, \sqrt{2}, \sqrt{3}\}$, 2

42. The set R of real numbers is :

(1) totally bounded (2) locally compact

(3) countably compact (4) sequentially compact

43. Every Lindelof metric space is :

(1) Compact (2) First countable

(3) Second countable (4) Reducible

44. Which of the following topology is coarser than the usual topology of R ?

(1) lower limit topology on R (2) upper limit topology on R

(3) co-countable topology on R (4) finite complement topology on R

45. Which of the following properties is Hereditary ?

(1) 2nd axiom of countablility (2) Compactness

(3) Lindelofness (4) Seperability

46. The concept of normality of a topological space was introduced by :

(1) Urysohn (2) Tichonov (3) Hausdorff (4) Tietze

47. Which of the following properties is ***not*** invariant under continuous map ?

(1) Lindeloffness (2) Separability

(3) 1st axiom of countability (4) Compactness

48. Which of the following statements is ***not correct*** ?

(1) Cantor set is perfect (2) Contor set is totally disconnected

(3) Cantor set is closed (4) Cantor set is countable

49. Let N be the set of non-negative integers. Then the collection

$H = \{F \; ; \; N - F \text{ is finite}\}$ is known as :

(1) Atomic Filter (2) Cofinite Filter

(3) Frechet Filter (4) Nbd Filter

50. Which of the following statement is ***not true*** ?

(1) Usual topological space (R, V) is Hausdorff

(2) Every indiscrete space containing at least two points is metrizable

(3) Every Discrete topological space is Hausdorff

(4) All metric spaces are Hausdorff

51. Which one of the following is ***not*** a topological property ?

(1) Boundedness (2) Compact (3) Closed (4) Open

52. Every metric space is paracompact. This theorem is named after :

(1) Stone (2) Michael (3) Lindelof (4) Hausdorff

53. Every convergent sequence in a topological space has a unique limit if X is :

(1) First countable Hausdorff space (2) T_1-space

(3) Hausdorff space (4) Second countable space

54. Regular spaces were first studied in 1921 by :

(1) Victoris (2) Hausdorff (3) Kolmogorov (4) Tietz

55. The result "A topological space is a Tychonoff space if and only if it is embeddable into a cube" is known as :

(1) Embedding Lemma (2) Tychonoff Embedding Theorem

(3) Urysohn's Metrization Theorem (4) None of these

56. The space $C[0, 1]$ is ***not*** a :

(1) Complete space (2) Normed linear space

(3) Metric space (4) Regular space

57. If (X, T) is an indiscrete topological space, then it has :

(1) no component (2) compact component

(3) finite number of components (4) only X as the component

58. For an empty set ϕ, which statement is ***true*** ?

(1) $d(\phi) = +\infty$ (2) $d(\phi) = -\infty$ (3) $\inf(\phi) = -\infty$ (4) none of these

59. Which of the following statement is ***not true*** ?

(1) R^n is connected (2) R is connected

(3) Q is connected (4) C^n is connected

60. The norm $||.||$ from a vector space X to R is a :

(1) Linear functional (2) Sublinear functional

(3) Bi-linear functional (4) Superlinear functional

61. The concept of reflexivity was introduced by :
(1) H. Hahn (2) F. Riesz (3) R. C. James (4) D. Hilbert

62. Which of the following is ***not*** a Hilbert space ?
(1) R^n (2) l_2 (3) $L_2[0, 1]$ (4) $L_1[0, 1]$

63. In a normed linear space, weak convergence implies strong convergence if :
(1) $\dim X < \infty$ (2) $\dim X > \infty$ (3) $\dim X = \infty$ (4) none of these

64. Which of the following is ***not true*** ?
(1) C as a real vector space is of dimension two
(2) $\mathbb{C}$ as a complex vector space is of dimension one
(3) l_1 is reflexive
(4) Space $C[a, b]$ is dense in $L_p[a, b]$

65. If x and y are orthonormal vectors in a Hilbert space H, then :
(1) $\|x-y\|=2$ (2) $\|x-y\|=\sqrt{2}$ (3) $\|x-y\|=0$ (4) $\|x-y\|=1$

66. L^p-spaces are complete. This result is known as :
(1) F. Riesz Theorem (2) Riesz Fisher Theorem
(3) Lebesgue Theorem (4) Jordan Decomposition Theorem

67. If P is a projection on a closed linear subspace M of H, then M is invariant under :
(1) $TP = PT$ (2) $P = TPT$ (3) $T = PTP$ (4) $TP = PTP$

68. If P is a projection on a Hilbert space H, then which of the following is ***not true*** ?
(1) P is a positive operator (2) $O \leq P \leq 1$
(3) $\|P\| > 1$ (4) $\|Px\| \leq \|x\| \quad \forall x \in H$

69. A one to one continuous linear transformation of a Banach space onto another Banach space is a :
(1) Homomorphism (2) Homeomorphism
(3) Closed Mapping (4) Open Mapping

70. A subspace Y of a Banach space Y is complete if and only if :
(1) The set Y is open in X (2) The set Y is complete in X
(3) The set Y is closed in X (4) None of the above

71. "A function $f(z)$ whose only singularities in the entire complex plane are poles" is called a :

(1) Analytic Function (2) Harmonic Function

(3) Entire Function (4) Meromorphic Function

72. Which of the following statement is ***not correct*** ?

(1) Subspace of Hausdorff space is Hausdorff

(2) Product of two Hausdorff spaces is Hausdorff

(3) The space X is Hausdorff if and only if the diagonal $\Delta = \{x \times x\, ; x \in X\}$ is open in XXX

(4) The space X is Hausdorff if and only if the diagonal $\Delta = \{x \times x\, ; x \in X\}$ is closed in XXX

73. The result "Let $X = A \cup B$ where A and B are closed in X. Let $f : A \to Y$ and $g : B \to Y$ be continuous. If $f(x) = g(x)$ for every $x \in A \cap B$, then f and g combine to give a continuous function $h : X \to Y$ defined by setting $h(x) = f(x)$ if $x \in A$ and $h(x) = g(x)$ if $x \in B$" is called :

(1) Pasting Lemma (2) Zorn's Lemma

(3) Embedding Lemma (4) Sequence Lemma

74. Every metric space is :

(1) Normed space (2) Paracompact

(3) Compact (4) Not first axiom sapce

75. If J is the Jacobian of functions u and v w.r.t. x and y and J_0 is the Jacobian of x and y. w.r.t. u and v, then :

(1) $JJ_0 = 1$ (2) $JJ_0 = 0$ (3) $JJ_0 = -1$ (4) $JJ_0 = 2$

76. Any infinite cyclic group has exactly k generators where :

(1) $k = 1$ (2) $k = 3$ (3) $k = 2$ (4) $k = 7$

77. The index of a saddle point is :

(1) 0 (2) 1 (3) -1 (4) does not exist

78. Let $F = \{f\}$ be an equicontinuous family of functions defined on a real interval I, then each function f is :

(1) continuous on I (2) uniformly continuous on I

(3) not continuous on I (4) constant on I

79. The critical point (0, 0) of the system $\frac{dx}{dt} = 4y, \frac{dy}{dt} = x$ is :

(1) stable
(2) asymptotically stable
(3) not stable
(4) stable but not asymptotically stable

80. Consider the linear autonomous system

$$\frac{dx}{dt} = ax + by, \frac{dy}{dt} = cx + dy$$

where a, b, c, d are real constants. If $a = d$ and b and c are of same sign such that $\sqrt{bc} < |a|$, then the critical point (0, 0) of the system is :

(1) saddle point (2) spiral point (3) node (4) centre

81. Solution of the I. V. P.

$$\frac{dy}{dx} = -y, y(0) = 1 \text{ is :}$$

(1) e^t (2) e^{-t} (3) $e^{-t/2}$ (4) $e^{t/2}$

82. Solution of the integral equation $\int_0^x e^{x-t} u(t) dt = x$ is :

(1) $x - 1$ (2) $x^2 - 1$ (3) $1 - x$ (4) x

83. The eigen values of the integral equation

$$u(x) = \lambda \int_{-1}^{1} (x + t) u(t) dt \text{ are :}$$

(1) $\pm \frac{\sqrt{3}}{2}$ (2) $\pm i \frac{\sqrt{3}}{2}$ (3) $\pm i\sqrt{3}$ (4) $1 \pm i\sqrt{3}$

84. If the homogeneous Fredholm integral equation :

$$u(x) = \lambda \int_a^b k(x, t) u(t) dt$$

has only a trivial solution, then the corresponding non-homogeneous equation has always :

(1) no solution
(2) Infinite number of solutions
(3) a unique solution
(4) only trivial solution

85. Which of the following theorem expresses the symmetric Kernel of a Fredholm integral equation as an infinite series of product of its orthogonal eigen functions ?

(1) Poincare Bendixon Theorem
(2) Bendixon Theorem
(3) Hilbert-Schmidt Theorem
(4) Mercer's Theorem

86. The problem of Brachistochrone (shortest time) was first formulated in the year 1696 by :

(1) Newton (2) Jeans Bernouli (3) Leibnitz (4) Jacques Bernouli

87. The curve which minimizes the functional $J(y) = \int_a^b (x-y)^2\, dx$ is :

(1) $x - y = 0$ (2) $x + y = 0$ (3) $x - 2y = 0$ (4) $y - 2x = 0$

88. The geodesics of the circular cylinder $\vec{r} = (a\cos\phi, a\sin\phi, z)$ is :

(1) Circle (2) Catenary (3) Straight line (4) Helix

89. In the Lipschitz condition $|f(t, y_1) - f(t, y_2)| \leq k|y_1 - y_2|$ condition on k is :

(1) $k > 0$ (2) $k \geq 0$ (3) $0 < k \leq 1$ (4) $k < 1$

90. If a rigid body rotates about a fixed point with an angular velocity $\vec{\omega}$ and has an angular momentum $\vec{H}$, then the kinetic energy T is given by :

(1) $\vec{\omega} \times \vec{H}$ (2) $\dfrac{\Delta.\vec{\omega}}{\vec{H}}$ (3) $\dfrac{1}{2}\vec{\omega}.\vec{H}$ (4) none of these

91. A condition is said to be steady-state if the dependent variables are :

(1) Not present in Heat equation (2) Independent of time t

(3) Dependent on time t (4) None of these

92. The one-dimensional wave equation for an elastic string of length L under boundary conditions $y(0, t) = 0$, $y(L, t) = 0$ indicates that :

(1) the string is not fixed at $x = 0$ (2) the string is only fixed at $x = 0$

(3) the string is fastened at both ends (4) none of these

93. If H represents Hamiltonian function, then $\dfrac{dH}{dt}$ is equal to :

(1) $\dfrac{\partial H}{\partial t}$ (2) $\dfrac{\partial^2 H}{\partial t^2}$ (3) $\dfrac{d^2 H}{dt^2}$ (4) None of these

94. The two dimensional Laplace equation in polar co-ordinates is given by :

(1) $\dfrac{\partial^2 u}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial u}{\partial r} = 0$ (2) $\dfrac{\partial^2 u}{\partial r^2} + \dfrac{\partial u}{\partial r} + \dfrac{1}{r}\dfrac{\partial^2 u}{\partial \theta^2} = 0$

(3) $\dfrac{\partial u}{\partial r} + \dfrac{1}{r}\dfrac{\partial^2 u}{\partial \theta^2} + \dfrac{1}{r^2}\dfrac{\partial^2 u}{\partial r^2} = 0$ (4) $\dfrac{\partial^2 u}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial u}{\partial r} + \dfrac{1}{r^2}\dfrac{\partial^2 u}{\partial \theta^2} = 0$

95. For the heat conduction equation $\frac{\partial u}{\partial t} = c\frac{\partial^2 u}{\partial x^2}$ in a bar subject to the boundary conditions that the end $x = 0$ is held at zero temperature and the end $x = 1$ is at temperature zero, the boundary conditions can be expressed at :

(1) $u(0, t) \neq 0; u(1, t) = 0$ (2) $u(1, t) \neq 0; u(0, t) = 0$

(3) $u(0, t) = 0; u(1, t) = 0$ (4) $u(0, t) \neq 0; u(1, t) \neq 0$

96. The boundary value problem which models the displacement function for a semi-infinite string which is initially undisturbed and is given an initial velocity is expressed as :

(1) $\frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$; $u(x, 0) \neq 0$

(2) $\frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$; $u(0, t) = 0$; $u(x, 0) = 0$

(3) $\frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$; $u(x, 0) = 0$; $\frac{\partial u}{\partial t}(x, 0) = 0$

(4) $\frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$; $u(x, 0) = 0$; $\frac{\partial u}{\partial t}(x, 0) = f(x)$

97. For the Lagrangian function $L(t, q_i, q_i)$ the generalized momenta p_i is defined as :

(1) $p_i = \frac{\partial L}{\partial q_i}$ (2) $p_i = \frac{\partial L}{\partial \dot{q}_i}$ (3) $p_i = \frac{\partial^2 L}{\partial q_i^2}$ (4) None of these

98. If a lead is sliding on a uniformly rotating wire in a force free space, then the equations of motion are :

(1) $\ddot{r} = rw^2$ (2) $\dot{r} = rw^2$ (3) $\ddot{r} = rw$ (4) $r = \ddot{r}w^2$

99. Principle of least action states that the variation of the Lagrange action W^* is zero for :

(1) the parabolic path (2) the circular path

(3) any path (4) the straight line path

100. Which one of the following form a set of Routh's equations ?

(1) $\frac{dq_\alpha}{dt} = \frac{\partial R}{\partial p_\alpha}, \frac{dp_\alpha}{dt} = -\frac{\partial R}{\partial q_\alpha}$ (2) $\frac{dq_\alpha}{dt} = -\frac{\partial R}{\partial p_\alpha}, \frac{dp_\alpha}{dt} = -\frac{\partial R}{\partial q_\alpha}$

(3) $\frac{dq_\alpha}{dt} = -\frac{\partial R}{\partial p_\alpha}, \frac{dp_\alpha}{dt} = \frac{\partial R}{\partial q_\alpha}$ (4) $\frac{dq_\alpha}{dt} = \frac{\partial R}{\partial p_\alpha} = -\frac{\partial R}{\partial q_\alpha}$

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