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MPHDURS-EE-2013
SUBJECT : Mathematics


18/9/13

C

10159

Sr. No.

Time : 1¼ Hours

Max. Marks : 100

Total Questions : 100

Candidate's Name _____ Date of Birth _____

Father's Name _____ Mother's Name _____

Roll No. (in figures) _____ (in words) _____

Date of Examination _____

(Signature of the Candidate)

(Signature of the Invigilator)

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- 1. All questions are compulsory and carry equal marks.**
- All the candidates **must return** the question booklet as well as OMR Answer-Sheet to the Invigilator concerned before leaving the Examination Hall, failing which a case of use of unfair-means/misbehaviour will be registered against him/her, in addition to lodging of an FIR with the police. Further the answer-sheet of such a candidate will not be evaluated.
- In case there is any discrepancy in any question(s) in the Question Booklet, the same may be brought to the notice of the Controller of Examinations in writing **within two hours** after the test is over. No such complaint(s) will be entertained thereafter.
- The candidate **must not** do any rough work or writing in the OMR Answer-Sheet. Rough work, if any, may be done in the question booklet itself. Answers **Should Not** be ticked in the question booklet.
- Use black or blue ball point pen only in the OMR Answer-Sheet.**
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MPHDURS-EE-2013/Mathematics/(C)

1. Which one of the following is *not* a topological property ?
(1) Boundedness (2) Compact (3) Closed (4) Open
2. Every metric space is paracompact. This theorem is named after :
(1) Stone (2) Michael (3) Lindelof (4) Hausdorff
3. Every convergent sequence in a topological space has a unique limit if X is :
(1) First countable Hausdorff space (2) T_1 -space
(3) Hausdorff space (4) Second countable space
4. Regular spaces were first studied in 1921 by :
(1) Victoris (2) Hausdorff (3) Kolmogorov (4) Tietz
5. The result "A topological space is a Tychonoff space if and only if it is embeddable into a cube" is known as :
(1) Embedding Lemma (2) Tychonoff Embedding Theorem
(3) Urysohn's Metrization Theorem (4) None of these
6. The space $C[0, 1]$ is *not* a :
(1) Complete space (2) Normed linear space
(3) Metric space (4) Regular space
7. If (X, T) is an indiscrete topological space, then it has :
(1) no component (2) compact component
(3) finite number of components (4) only X as the component
8. For an empty set ϕ , which statement is *true* ?
(1) $d(\phi) = +\infty$ (2) $d(\phi) = -\infty$ (3) $\inf(\phi) = -\infty$ (4) none of these
9. Which of the following statement is *not true* ?
(1) R^n is connected (2) R is connected
(3) Q is connected (4) C^n is connected
10. The norm $|| \cdot ||$ from a vector space X to R is a :
(1) Linear functional (2) Sublinear functional
(3) Bi-linear functional (4) Superlinear functional

11. The basis and the degree of the extension $Q(\sqrt{2}, \sqrt{3})$ over Q is :
- | | |
|--|------------------------------------|
| (1) $\{\sqrt{2}, \sqrt{3}\}, 4$ | (2) $\{1, \sqrt{2}, \sqrt{3}\}, 4$ |
| (3) $\{1, \sqrt{2}, \sqrt{3}, \sqrt{6}\}, 4$ | (4) $\{1, \sqrt{2}, \sqrt{3}\}, 2$ |
12. The set R of real numbers is :
- | | |
|-----------------------|--------------------------|
| (1) totally bounded | (2) locally compact |
| (3) countably compact | (4) sequentially compact |
13. Every Lindelof metric space is :
- | | |
|----------------------|---------------------|
| (1) Compact | (2) First countable |
| (3) Second countable | (4) Reducible |
14. Which of the following topology is coarser than the usual topology of R ?
- | | |
|----------------------------------|---------------------------------------|
| (1) lower limit topology on R | (2) upper limit topology on R |
| (3) co-countable topology on R | (4) finite complement topology on R |
15. Which of the following properties is Hereditary ?
- | | |
|-------------------------------|------------------|
| (1) 2nd axiom of countability | (2) Compactness |
| (3) Lindelofness | (4) Separability |
16. The concept of normality of a topological space was introduced by :
- | | |
|---------------|--------------|
| (1) Urysohn | (2) Tichonov |
| (3) Hausdorff | (4) Tietze |
17. Which of the following properties is *not* invariant under continuous map ?
- | | |
|-------------------------------|------------------|
| (1) Lindeloffness | (2) Separability |
| (3) 1st axiom of countability | (4) Compactness |
18. Which of the following statements is *not correct* ?
- | | |
|---------------------------|--|
| (1) Cantor set is perfect | (2) Cantor set is totally disconnected |
| (3) Cantor set is closed | (4) Cantor set is countable |
19. Let N be the set of non-negative integers. Then the collection
- $$H = \{F ; N - F \text{ is finite}\}$$
- is known as :
- | | |
|--------------------|---------------------|
| (1) Atomic Filter | (2) Cofinite Filter |
| (3) Frechet Filter | (4) Nbd Filter |

20. Which of the following statement is *not true* ?
(1) Usual topological space (R, V) is Hausdorff
(2) Every indiscrete space containing at least two points is metrizable
(3) Every Discrete topological space is Hausdorff
(4) All metric spaces are Hausdorff
21. The product of two odd permutations is :
(1) even and odd (2) odd (3) even (4) none of these
22. A group has almost one composition series. This result is known as :
(1) Cayley's theorem (2) Sylow's theorem
(3) Lagrange's theorem (4) Jordan-Holder theorem
23. If every non-constant polynomial over a field F has all its roots in F , then F is :
(1) Algebraically Closed Field (2) Prime Field
(3) Perfect Field (4) None of the above
24. Let $R = F[x]$ be a polynomial ring over a field F . Then R is :
(1) Artinian but not Noetherian (2) Artinian and Noetherian both
(3) Neither Artinian nor Noetherian (4) Noetherian but not Artinian
25. Which of the following is a prime field ?
(1) \mathbb{Q} (2) \mathbb{R} (3) \mathbb{C} (4) \mathbb{Z}_n
26. Let G be a commutative group having composition series. Then G must be :
(1) Infinite (2) Finite
(3) Finite with $G' = G$ (4) Infinite with $Z(G) = \langle e \rangle$
27. Let M be a simple R -module and $T \in \text{Home}_R(M, M)$ such that $T \neq 0$, then :
(1) $I_m(T) = O$ (2) $\ker(T) = M$ (3) T is singular (4) T is non-singular
28. A composition series for a group is :
(1) Central series (2) Derived series
(3) Solvable series (4) None of these

29. The degree of the splitting field of the polynomial $f(x) = x^{10} - 1$ over Q is :
 (1) 10 (2) 4 (3) 6 (4) 8
30. Any group of order 15 is :
 (1) Abelian (2) Simple (3) Cyclic (4) p -group
31. If a function $f(z)$ is analytic except at finite number of singularities (including that at infinity), then the sum of residues of these singularities is :
 (1) $2\pi i$ (2) πi (3) finite (4) zero
32. The transformation $f(z) = x - iy$ is :
 (1) analytic (2) conformal (3) isogonal (4) none of these
33. The set of all bilinear transformation under the product of transformations form a :
 (1) Monoid (2) Abelian group (3) Semi group (4) Non-Abelian group
34. The function $f(z) = e^{1/z}$ has essential singularity at :
 (1) $Z = 1$ (2) $Z = 0$ (3) $Z = 2$ (4) $Z = -1$
35. Which of the following statement is *not true* ?
 (1) Exponential function is analytic
 (2) Absolute value function when defined on the set of real or complex numbers is analytic
 (3) Power functions are analytic
 (4) Any polynomial is an analytic function
36. The simple poles of Gamma function are at :
 (1) $Z = 0, 1, 2, \dots, n, \dots$ (2) $Z = 0, -1, -2, \dots, -n, \dots$
 (3) $Z = 1, 2, \dots, n, \dots$ (4) None of these
37. If $f(z)$ and $g(z)$ are analytic inside and on a closed contour and $|g(z)| < |f(z)|$ on C , then $f(z)$ and $f(z) + g(z)$ have same :
 (1) value (2) number of poles
 (3) number of singularities (4) number of zeros
38. The residue of $f(z) = \frac{z^3}{z^2 - 1}$ at $z = \infty$ is :
 (1) -1 (2) 1 (3) 0 (4) 3

46. Which of the following is *not true* ?

- (1) Every absolutely continuous function is of bounded variation
- (2) Every bounded function is of bounded variation
- (3) Every monotone function on $[a, b]$ is of bounded variation
- (4) Every function of bounded variation is bounded

47. The word "Topologi" was introduced in Germany in 1847 by :

- (1) George Cantor
- (2) Johann Benedict
- (3) Kazimierz Kuratowski
- (4) Felix Hausdorff

48. A function which is analytic for all finite values of Z and bounded is :

- (1) a constant
- (2) zero
- (3) a function of Z
- (4) continuous

49. The result "The order of a canonical product is equal to the exponent of convergence of its zeros" is known as :

- (1) Borel's theorem
- (2) Jensen's formula
- (3) Bloch's theorem
- (4) Morera's theorem

50. The constant

$$r = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} - \log n \right)$$

is called :

- (1) Euler's constant
- (2) Euler's number
- (3) Lebesgue constant
- (4) Lebesgue number

51. The function $D:R \rightarrow R$ such that

$$D(x) = \begin{cases} 1 & \text{if } x \in Q \\ 0 & \text{if } x \notin Q \end{cases}$$

is known as :

- (1) Step Function
- (2) Simple Function
- (3) Characteristic Function
- (4) Dirichlet's Function

52. Every convergent sequence of measurable functions is nearly uniformly convergent. This result is known as :

- (1) 1st principle of measurability
- (2) Littlewood's 2nd principle of measurability
- (3) Littlewood's third principle
- (4) Egorov's theorem

53. If a_n and b_n are sequences of extended real numbers and $a_n \leq b_n$ for all n sufficiently large. Which of the following is *not true* ?
- (1) $\liminf a_n \geq \liminf b_n$ (2) $\liminf a_n \leq \liminf b_n$
(3) $\limsup a_n \leq \limsup b_n$ (4) None of these
54. The composition of two Lebesgue measurable functions is :
- (1) not necessarily Lebesgue measurable
(2) Borel measurable
(3) always measurable
(4) always Lebesgue measurable
55. Every uniformly continuous function is :
- (1) Absolutely continuous (2) Not absolutely continuous
(3) Not Continuous (4) None of these
56. Which of the following statements is *not correct* ?
- (1) Ch. function of irrational numbers in $[0, 1]$ is Riemann integrable
(2) Ch. functions are simple functions
(3) Ch. function of the set E of rational numbers in $[0, 1]$ is measurable
(4) None of the above
57. Let A be the set of algebraic numbers. Then the outer measure of A is equal to :
- (1) ∞ (2) a finite measure
(3) zero (4) outer measure of the set of real numbers
58. The axiom of choice was formulated in 1904 by :
- (1) Riemann (2) Ernst Zermelo (3) G. H. Moore (4) George Cantor
59. The result "Let $(-1, 1)$ be interval of convergence for the power series $\sum a_n x^n$. If $\sum_{n=0}^{\infty} a_n = S$, then $\lim_{x \rightarrow 1-0} \sum_{n=0}^{\infty} a_n x^n = S$ " is known as :
- (1) Uniqueness theorem (2) Weierstrass's theorem
(3) Tauber's theorem (4) Abel's theorem
60. If a function f is convex and $f(0) \leq 0$, then :
- (1) f is superadditive on the positive half axis
(2) f is additive
(3) f is subadditive on the positive half axis
(4) f is superconvex

61. A condition is said to be steady-state if the dependent variables are :
- (1) Not present in Heat equation (2) Independent of time t
 (3) Dependent on time t (4) None of these
62. The one-dimensional wave equation for an elastic string of length L under boundary conditions $y(0, t) = 0, y(L, t) = 0$ indicates that :
- (1) the string is not fixed at $x = 0$ (2) the string is only fixed at $x = 0$
 (3) the string is fastened at both ends (4) none of these
63. If H represents Hamiltonian function, then $\frac{dH}{dt}$ is equal to :
- (1) $\frac{\partial H}{\partial t}$ (2) $\frac{\partial^2 H}{\partial t^2}$ (3) $\frac{d^2 H}{dt^2}$ (4) None of these
64. The two dimensional Laplace equation in polar co-ordinates is given by :
- (1) $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} = 0$ (2) $\frac{\partial^2 u}{\partial r^2} + \frac{\partial u}{\partial r} + \frac{1}{r} \frac{\partial^2 u}{\partial \theta^2} = 0$
 (3) $\frac{\partial u}{\partial r} + \frac{1}{r} \frac{\partial^2 u}{\partial \theta^2} + \frac{1}{r^2} \frac{\partial^2 u}{\partial r^2} = 0$ (4) $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2} = 0$
65. For the heat conduction equation $\frac{\partial u}{\partial t} = c \frac{\partial^2 u}{\partial x^2}$ in a bar subject to the boundary conditions that the end $x = 0$ is held at zero temperature and the end $x = 1$ is at temperature zero, the boundary conditions can be expressed at :
- (1) $u(0, t) \neq 0; u(1, t) = 0$ (2) $u(1, t) \neq 0; u(0, t) = 0$
 (3) $u(0, t) = 0; u(1, t) = 0$ (4) $u(0, t) \neq 0; u(1, t) \neq 0$
66. The boundary value problem which models the displacement function for a semi-infinite string which is initially undisturbed and is given an initial velocity is expressed as :
- (1) $\frac{1}{c^2} \frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}; u(x, 0) \neq 0$
 (2) $\frac{1}{c^2} \frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}; u(0, t) = 0; u(x, 0) = 0$
 (3) $\frac{1}{c^2} \frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}; u(x, 0) = 0; \frac{\partial u}{\partial t}(x, 0) = 0$
 (4) $\frac{1}{c^2} \frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}; u(x, 0) = 0; \frac{\partial u}{\partial t}(x, 0) = f(x)$

67. For the Lagrangian function $L(t, q_i, \dot{q}_i)$ the generalized momenta p_i is defined as :

(1) $p_i = \frac{\partial L}{\partial \dot{q}_i}$ (2) $p_i = \frac{\partial L}{\partial q_i}$ (3) $p_i = \frac{\partial^2 L}{\partial q_i^2}$ (4) None of these

68. If a lead is sliding on a uniformly rotating wire in a force free space, then the equations of motion are :

(1) $\ddot{r} = r\omega^2$ (2) $\dot{r} = r\omega^2$ (3) $\ddot{r} = r\omega$ (4) $r = \dot{r}\omega^2$

69. Principle of least action states that the variation of the Lagrange action W^* is zero for :

- (1) the parabolic path (2) the circular path
(3) any path (4) the straight line path

70. Which one of the following form a set of Routh's equations ?

(1) $\frac{dq_\alpha}{dt} = \frac{\partial R}{\partial p_\alpha}, \frac{dp_\alpha}{dt} = -\frac{\partial R}{\partial q_\alpha}$ (2) $\frac{dq_\alpha}{dt} = -\frac{\partial R}{\partial p_\alpha}, \frac{dp_\alpha}{dt} = -\frac{\partial R}{\partial q_\alpha}$
(3) $\frac{dq_\alpha}{dt} = -\frac{\partial R}{\partial p_\alpha}, \frac{dp_\alpha}{dt} = \frac{\partial R}{\partial q_\alpha}$ (4) $\frac{dq_\alpha}{dt} = \frac{\partial R}{\partial p_\alpha} = -\frac{\partial R}{\partial q_\alpha}$

71. Solution of the I. V. P.

$$\frac{dy}{dx} = -y, y(0) = 1 \text{ is :}$$

(1) e^t (2) e^{-t} (3) $e^{-t/2}$ (4) $e^{t/2}$

72. Solution of the integral equation $\int_0^x e^{x-t} u(t) dt = x$ is :

(1) $x - 1$ (2) $x^2 - 1$ (3) $1 - x$ (4) x

73. The eigen values of the integral equation

$$u(x) = \lambda \int_{-1}^1 (x+t)u(t) dt \text{ are :}$$

(1) $\pm \frac{\sqrt{3}}{2}$ (2) $\pm i \frac{\sqrt{3}}{2}$ (3) $\pm i\sqrt{3}$ (4) $1 \pm i\sqrt{3}$

74. If the homogeneous Fredholm integral equation :

$$u(x) = \lambda \int_a^b k(x,t) u(t) dt$$

has only a trivial solution, then the corresponding non-homogeneous equation has always :

- (1) no solution (2) Infinite number of solutions
(3) a unique solution (4) only trivial solution

75. Which of the following theorem expresses the symmetric Kernel of a Fredholm integral equation as an infinite series of product of its orthogonal eigen functions ?
- (1) Poincare Bendixon Theorem (2) Bendixon Theorem
 (3) Hilbert-Schmidt Theorem (4) Mercer's Theorem
76. The problem of Brachistochrone (shortest time) was first formulated in the year 1696 by :
- (1) Newton (2) Jeans Bernouli (3) Leibnitz (4) Jacques Bernouli
77. The curve which minimizes the functional $J(y) = \int_a^b (x-y)^2 dx$ is :
- (1) $x - y = 0$ (2) $x + y = 0$ (3) $x - 2y = 0$ (4) $y - 2x = 0$
78. The geodesics of the circular cylinder $\vec{r} = (a \cos \phi, a \sin \phi, z)$ is :
- (1) Circle (2) Catenary (3) Straight line (4) Helix
79. In the Lipschitz condition $|f(t, y_1) - f(t, y_2)| \leq k |y_1 - y_2|$ condition on k is :
- (1) $k > 0$ (2) $k \geq 0$ (3) $0 < k \leq 1$ (4) $k < 1$
80. If a rigid body rotates about a fixed point with an angular velocity $\vec{\omega}$ and has an angular momentum \vec{H} , then the kinetic energy T is given by :
- (1) $\vec{\omega} \times \vec{H}$ (2) $\frac{\Delta \cdot \vec{\omega}}{\vec{H}}$
 (3) $\frac{1}{2} \vec{\omega} \cdot \vec{H}$ (4) none of these
81. "A function $f(z)$ whose only singularities in the entire complex plane are poles" is called a :
- (1) Analytic Function (2) Harmonic Function
 (3) Entire Function (4) Meromorphic Function
82. Which of the following statement is **not correct** ?
- (1) Subspace of Hausdorff space is Hausdorff
 (2) Product of two Hausdorff spaces is Hausdorff
 (3) The space X is Hausdorff if and only if the diagonal $\Delta = \{x \times x; x \in X\}$ is open in XXX
 (4) The space X is Hausdorff if and only if the diagonal $\Delta = \{x \times x; x \in X\}$ is closed in XXX

83. The result "Let $X = A \cup B$ where A and B are closed in X . Let $f : A \rightarrow Y$ and $g : B \rightarrow Y$ be continuous. If $f(x) = g(x)$ for every $x \in A \cap B$, then f and g combine to give a continuous function $h : X \rightarrow Y$ defined by setting $h(x) = f(x)$ if $x \in A$ and $h(x) = g(x)$ if $x \in B$ " is called :

- (1) Pasting Lemma (2) Zorn's Lemma
 (3) Embedding Lemma (4) Sequence Lemma

84. Every metric space is :

- (1) Normed space (2) Paracompact
 (3) Compact (4) Not first axiom sapce

85. If J is the Jacobian of functions u and v w.r.t. x and y and J_0 is the Jacobian of x and y w.r.t. u and v , then :

- (1) $JJ_0 = 1$ (2) $JJ_0 = 0$ (3) $JJ_0 = -1$ (4) $JJ_0 = 2$

86. Any infinite cyclic group has exactly k generators where :

- (1) $k = 1$ (2) $k = 3$ (3) $k = 2$ (4) $k = 7$

87. The index of a saddle point is :

- (1) 0 (2) 1
 (3) -1 (4) does not exist

88. Let $F = \{f\}$ be an equicontinuous family of functions defined on a real interval I , then each function f is :

- (1) continuous on I (2) uniformly continuous on I
 (3) not continuous on I (4) constant on I

89. The critical point $(0, 0)$ of the system $\frac{dx}{dt} = 4y, \frac{dy}{dt} = x$ is :

- (1) stable (2) asymptotically stable
 (3) not stable (4) stable but not asymptotically stable

90. Consider the linear autonomous system

$$\frac{dx}{dt} = ax + by, \frac{dy}{dt} = cx + dy$$

where a, b, c, d are real constants. If $a = d$ and b and c are of same sign such that $\sqrt{bc} < |a|$, then the critical point $(0, 0)$ of the system is :

- (1) saddle point (2) spiral point (3) node (4) centre

91. The concept of reflexivity was introduced by :
 (1) H. Hahn (2) F. Riesz (3) R. C. James (4) D. Hilbert
92. Which of the following is *not* a Hilbert space ?
 (1) R^n (2) l_2 (3) $L_2[0, 1]$ (4) $L_1[0, 1]$
93. In a normed linear space, weak convergence implies strong convergence if :
 (1) $\dim X < \infty$ (2) $\dim X > \infty$ (3) $\dim X = \infty$ (4) none of these
94. Which of the following is *not true* ?
 (1) C as a real vector space is of dimension two
 (2) \mathcal{C} as a complex vector space is of dimension one
 (3) l_1 is reflexive
 (4) Space $C[a, b]$ is dense in $L_p[a, b]$
95. If x and y are orthonormal vectors in a Hilbert space H , then :
 (1) $\|x - y\| = 2$ (2) $\|x - y\| = \sqrt{2}$ (3) $\|x - y\| = 0$ (4) $\|x - y\| = 1$
96. L^p -spaces are complete. This result is known as :
 (1) F. Riesz Theorem (2) Riesz Fisher Theorem
 (3) Lebesgue Theorem (4) Jordan Decomposition Theorem
97. If P is a projection on a closed linear subspace M of H , then M is invariant under :
 (1) $TP = PT$ (2) $P = TPT$ (3) $T = PTP$ (4) $TP = PTP$
98. If P is a projection on a Hilbert space H , then which of the following is *not true* ?
 (1) P is a positive operator (2) $0 \leq P \leq 1$
 (3) $\|P\| > 1$ (4) $\|Px\| \leq \|x\| \quad \forall x \in H$
99. A one to one continuous linear transformation of a Banach space onto another Banach space is a :
 (1) Homomorphism (2) Homeomorphism
 (3) Closed Mapping (4) Open Mapping
100. A subspace Y of a Banach space X is complete if and only if :
 (1) The set Y is open in X (2) The set Y is complete in X
 (3) The set Y is closed in X (4) None of the above

SEAT