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M.Phil./Ph.D./URS-EE-Oct.-2017

SUBJECT : Mathematics

A

10217

Sr. No. ....

Time : **1¼ Hours** Max. Marks : **100** Total Questions : **100**

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SEAL

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1. If $n(A) = 115$, $n(B) = 225$, $n(A - B) = 73$, then $n(A \cup B) =$

(1) 265 (2) 278 (3) 295 (4) 298

2. The set of interior points of which of the following sets is ***not*** empty ?

(1) R (2) N (3) Z (4) I

3. If X is the set of even natural numbers less than 8 and Y is the set of odd prime numbers less than equal to 7, then the number of relations from X to Y is :

(1) 9 (2) $2^9 - 1$ (3) 2^{9-1} (4) 2^9

4. The sequence $S_n = \begin{cases} 2 & , \text{ when } n \text{ is even} \\ \text{lowest prime factor } (\neq 1) \text{ of } n & , \text{ when } n \text{ is odd} \end{cases}$ has limit point :

(1) 2

(2) countable in number

(3) 1, 2, 3, 4, ...

(4) uncountable in number

5. The series $\sum_{n=1}^{\infty} \frac{1}{\left(1+\frac{1}{n}\right)^{n^2}}$ is :

(1) convergent

(2) conditionally convergent

(3) divergent

(4) oscillatory

6. $\lim_{n\to\infty}\left[\frac{n}{n^2}+\frac{n}{(n^2+1^2)}+\frac{n}{(n^2+2^2)}+......\right]=$

(1) 0 (2) $\frac{\pi}{3}$ (3) $\frac{\pi}{4}$ (4) $\frac{-\pi}{4}$

7. $\lim_{x\to 0}\frac{\sinh x - \sin x}{x \sin^2 x} =$

(1) $\frac{1}{6}$ (2) $\frac{1}{4}$ (3) $\frac{1}{3}$ (4) $\frac{1}{2}$

8. Let $f(x)=\begin{cases}\dfrac{e^{1/x}}{1+e^{1/x}} & , \quad x\neq 0\\ 0 & , \quad x=0\end{cases}$, then which of the following is true ?

(1) $f(x)$ is continuous at $x = 0$

(2) $f(x)$ is discontinuous at $x = 0$

(3) $f(x)$ is differentiable at $x = 0$

(4) $f(x)$ has discontinuity of first kind from left at $x = 0$

9. If $f(x + y) = f(x)\, f(y)\ \forall\ x, y$ and $f(5) = -2$ and $f'(0) = 3$, then the value of $f'(5) =$

(1) -3 (2) -5 (3) -6 (4) 6

10. What is the abscissa of the point at which the tangent to the curve $y = x\,(x - 1)$ is parallel to the chord joining the extrimities of the curve in the interval [1, 2] ?

(1) 5/4 (2) 5/3 (3) 4/3 (4) 3/2

11. A function $f(x)$ is a monotonic function if $f(x)$ is :

(1) either increasing or decreasing function

(2) only increasing function

(3) only decreasing function

(4) a constant function

12. The integral $\int_0^\infty \frac{\sin x}{x}\, dx$:

(1) converges absolutely (2) does not converge

(3) converges but not absolutely (4) does not exist

13. Let $f_n(x) = nxe^{-nx^2}, x \in [0,1]$, then which of the following is ***not*** a point sequence ?

(1) $a_n = 2ne^{-2n}$ (2) $a_n = ne^{-n}$ (3) $a_n = 2ne^{-4n}$ (4) $a_n = 0$

14. If $f(x,y)=\begin{cases}\dfrac{xy^2}{x^2+y^4} & , (x,y)\neq(0,0)\\ 0 & , (x,y)=(0,0)\end{cases}$, then the directional derivative along $u=\left(\sqrt{2},\sqrt{2}\right)$ at (0, 0) is :

(1) $\sqrt{2}$ (2) $\dfrac{1}{\sqrt{2}}$ (3) $2\sqrt{2}$ (4) $\dfrac{1}{2\sqrt{2}}$

15. If $f: X \to R$, $X \subseteq R^2$ and $(a, b) \in X$ is such that f_x, f_y are differentiable at (a, b), then $f_{xy}(a,b)=f_{yx}(a,b)$. This result is known as :

(1) Schwarz's theorem (2) Young's theorem

(3) Taylor's theorem (4) Implicit function theorem

16. The function $f(x,y)=x^4+x^2y+y^2$ is :

(1) minimum at (0, 0)

(2) neither minimum nor maximum at (0, 0)

(3) maximum at (0, 0)

(4) discontinuous at (0, 0)

17. If $< E_i >$ is a sequence of Lebesgue measurable sets and m is the Lebesgue measure, then :

(1) $m(\cup E_i)=\sum mE_i$ (2) $m(\cup E_i)\geq\sum mE_i$

(3) $m(\cup E_i)\leq\sum mE_i$ (4) $m(\cup E_i)=0$

18. If $\alpha > 0$ and $\beta > 0$ and $f(x)=\begin{cases}x^{\alpha}\sin\dfrac{1}{x^{\beta}} & , x\neq 0\\ 0 & , x=0\end{cases}$, then $f(x)$ is of bounded variation in [0, 1] if :

(1) $\alpha < \beta$ (2) $1+\alpha < \beta$ (3) $\alpha+\beta=1$ (4) $\alpha > \beta$

19. Let $X = \{x : 0 < d(0, x) \leq 1$, and $x \in R^2\}$ where $0 = (0, 0)$ and d is the usual metric on X, then which of the following is ***not*** true ?

(1) X is closed (2) X is bounded

(3) X is not compact (4) X is compact

20. If $X = [X_1\ X_2 ......X_n]'$ is an n-tuple non-zero vector, then the $n \times n$ matrix $V = XX'$:

(1) has rank zero (2) has rank 1 (3) has rank n (4) is orthogonal

21. For the matrix $A = \begin{bmatrix} 2 & -2 & 3 \\ -2 & -1 & 6 \\ 1 & 2 & 0 \end{bmatrix}$, one of the eigen values is 3, the other two eigen values are :

(1) 2, –5 (2) 3, 5 (3) 3, –5 (4) 2, 5

22. Let V be the vector space of ordered pairs of complex numbers over the real field R, then the dimension of V is :

(1) 6 (2) 4 (3) 2 (4) 1

23. Let T be a linear transformation on a vector space V such that $T^2 - T + 1 = 0$, then T is :

(1) singular (2) invertible (3) not invertible (4) idempotent

24. A real quadratic form $X^T AX$ in three variables is equivalent to the diagonal form $3y_1^2 - 4y_2^2 + 5y_3^2$. Then, the quadratic form $X^T AX$ is :

(1) positive definite (2) negative definite

(3) positive semi-definite (4) indefinite

25. The orthogonal complement of inner product space V is :

(1) zero subspace (2) any subspace

(3) V itself (4) None of these

26. If $z = -\sqrt{3} - i$, then a value of z^4 is :

(1) $8(1 + i\sqrt{3})$ (2) $8(1 - i\sqrt{3})$ (3) $8(-1 + i\sqrt{3})$ (4) $8(\sqrt{3} - i)$

27. If $w = u + iv$, $z = x + iy$, then the image of the line $x = -3$ under the complex mapping $w = z^2$ is :

(1) $u = -3v$ (2) $u = -3 + \frac{v^2}{4}$ (3) $v = 9 - \frac{u^2}{36}$ (4) $u = 9 - \frac{v^2}{36}$

28. Value of $i^{1/3}$ is :

(1) $\frac{\sqrt{3}}{2} - \frac{1}{2}i$ (2) $\frac{\sqrt{3}}{2} + \frac{1}{2}i$ (3) $\frac{1}{2} + \frac{\sqrt{3}}{2}i$ (4) $\frac{1}{2} + \frac{1}{2}i$

29. Solution of $e^{z-1} = -ie^3$ is :

(1) $4 + (2n-1)\pi i$

(2) $4 + \frac{1}{2}(2n-1)\pi i$

(3) $4 + \frac{1}{2}(4n-1)\pi i$

(4) $\frac{1}{4}(2n-1)\pi i$

30. Solution of the equation $\cos z = i \sin z$ is :

(1) $z = \frac{2(n+1)}{3}\pi i$

(2) $z = (4n+1)\frac{\pi i}{2}$

(3) $\left(\frac{2n+1}{2}\right)\pi i$

(4) No solution

31. The function $f(z) = \overline{z}$ is :

(1) Everywhere differentiable

(2) Nowhere differentiable

(3) Differentiable only at $z = 0$

(4) Differentiable everywhere except at $z = 0$

32. $\int_r \frac{1}{z}\, dz$, where $r(t) = \sin t + i \cos t,\ 0 \le t \le 2\pi$, is :

(1) -2π

(2) $4\pi i$

(3) $2\pi i$

(4) $-2\pi i$

33. Using Cauchy's integral formula for derivatives $\int_C \frac{\sin z}{z^4}\, dz$, where C is the circle $|z| = 2$, is :

(1) $\pi i/2$

(2) $\pi i/3$

(3) $-\pi i/3$

(4) $-\pi i/2$

34. For the function $f(z) = z^3 \sin\left(\frac{1}{z}\right)$, the point $z = 0$ is :

(1) zero of order one

(2) pole of order one

(3) essential singularity

(4) zero of order two

35. Residue of $f(z) = e^{3/z}$ at $z = 0$, is :

(1) $\frac{1}{3}$

(2) 3

(3) e^3

(4) ∞

36. Using Cauchy Residue theorem, $\oint_C \frac{2z+6}{z^2+4}dz$, where C is $|z-i| = 2$, is :

(1) $\frac{\pi}{2}(2+3i)$ (2) $\frac{\pi}{2}(3+2i)$ (3) $\pi(2+3i)$ (4) $\pi(3+2i)$

37. The mapping $f(z) = ze^{z^2-2}$ is not conformal at $z =$

(1) $\pm\frac{i}{\sqrt{2}}$ (2) $\pm\frac{1}{\sqrt{2}}$ (3) $\pm\sqrt{2}i$ (4) 0

38. If 20 dictionaries in a library contain 41727 pages, then one of the dictionaries must have at least :

(1) 2084 pages (2) 2085 pages (3) 2086 pages (4) 2087 pages

39. In how many ways can a person can invite his 5 friends for dinner ?

(1) 31 (2) 32 (3) 33 (4) 120

40. For any integer $n > 2$, which of the following is true for the Euler's function $\phi(n)$?

(1) $\phi(n)$ is zero (2) $\phi(n)$ is even

(3) $\phi(n)$ is odd (4) $\phi(n)$ is rational number

41. The generators of the group $G = \{a, a^2, a^3, a^4, a^5, a^6 = e\}$ are :

(1) a and a^5 (2) a^3 and a^5 (3) a^2 and a^3 (4) a and a^3

42. If N is the set of natural numbers, then under the binary operation $a \,.\, b = a + b$, $(N, .)$ is :

(1) group (2) semi-group (3) quasi-group (4) monoid

43. If C = (1, 2, 3, 4), then C^2 is :

(1) (1, 4) (2, 3) (2) (1, 3) (3) (1, 3) (2, 4) (4) (2, 4)

44. Which of the following is *not* true ?

The relation of isomorphism in the set of all groups :

(1) satisfies reflexivity (2) satisfies anti-symmetry

(3) satisfies transitivity (4) is an equivalence relation

45. $\left[Q\left(\sqrt{2}, \sqrt{3}\right) : Q\right] =$

(1) 4 (2) 3 (3) 2 (4) 1

46. Which of the following is ***not*** a field ?

(1) $\frac{z}{2z}$ (2) $\frac{z}{3z}$ (3) $\frac{z}{4z}$ (4) $\frac{z}{5z}$

47. Given a field F and the set M of all 2×2 matrices of the form $\begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix}$ for $a, b \in F$, then which of the following is ***not*** true ?

(1) M is left ideal in F

(2) M is right ideal in F

(3) M is subring of F

(4) M is right ideal but not left ideal in F

48. Which of the following is ***not*** true ?

(1) Every metric space is first countable

(2) A metric space is second countable iff it is separable

(3) Every metric space is second countable

(4) Any open subspace of a separable space is separable

49. Which of the following is ***not*** correct ?

(1) Every compact metric space is complete

(2) A compact Hausdorff space is normal

(3) A compact subspace of a Hausdorff space is closed

(4) Every metric space is compact Hausdorff space

50. If $X = \{a, b, c\}$, $T = \{\phi, X, \{a, c\}, \{b\}\}$, then the topological space (X, T) is :

(1) not a connected space

(2) A connected space

(3) not a Hausdorff space

(4) not a compact space

51. Solution of $\frac{dy}{dx}=\frac{y}{x}+x\tan\frac{y}{x}$ is :

(1) $\log\cot\frac{y}{x}=x+c$

(2) $\log\tan\frac{y}{x}=x+c$

(3) $\log\cot\frac{y}{2x}=x+c$

(4) $\log\tan\frac{y}{2x}=x+c$

52. Solution of $y=xp-p^2$ is :

(1) $y=\log x+c$ (2) $y=cx-c^2$ (3) $y=3x-c$ (4) $y=cx-cx^2$

53. P. I. of $(D^2-6D+9)y=8e^{3x}$ is :

(1) $4xe^{3x}$ (2) $\frac{x}{2}e^{3x}$ (3) $2x^2e^{3x}$ (4) $4x^2e^{3x}$

54. Solving $y''-2y'+y=e^x\log x$ by variation for parameters, the value of Wronskion is :

(1) e^{-2x} (2) e^{2x} (3) xe^{2x} (4) $x\bar{e}^{2x}$

55. Green's function of the boundary value problem $y''=0, y(0)=y(1)=0$ is given by :

(1) $G(x, t) = x(1-t); 0 \le x < t$

(2) $G(x, t) = x\,t; t < x \le 1$

(3) $G(x, t) = x^2(1-t); 0 \le x < t$

(4) $G(x, t) = x(1-t^2); t < x \le 1$

56. The general solution of $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ is of the form :

(1) $u=cf(x-iy)$

(2) $u=cf(x+y)$

(3) $u=f(x+y)+g(x-y)$

(4) $u=f(x+iy)+g(x-iy)$

57. The initial value problem $u_x+u_y=1, u(s,s)=\sin s, 0\le s\le 1$ has :

(1) no solution

(2) a unique solution

(3) two solutions

(4) infinitely many solutions

58. The region in which the differential equation $yu_{xx} + 2xyu_{xy} + xu_{yy} = u_x + u_y$, is hyperbolic, is :

(1) $xy > 0$ (2) $xy \neq 0$ (3) $xy > 1$ (4) $xy \neq 1$

59. The partial differential equation formed by eliminating arbitrary function from the equation $z = f\left(x^2 - y^2\right)$ is :

(1) $xp + yq = 0$ (2) $xq + yp = 0$ (3) $\frac{x}{y} = q$ (4) $\frac{x}{y} = p$

60. P. I. of $\left(2D^2 - 3DD' + D'^2\right)z = e^{x+2y}$ is :

(1) $x^2 e^{x+2y}$ (2) xe^{x+2y} (3) $-\frac{x}{3}e^{x+2y}$ (4) $-\frac{x}{2}e^{x+2y}$

61. $\left(\frac{\Delta^2}{E}\right)e^x \cdot \frac{Ee^x}{\Delta^2 e^x} =$

(1) e^x (2) xe^x (3) $(x+1)e^x$ (4) $\frac{e^x}{x}$

62. Consider the series $x_{n+1} = \frac{x_n}{2} + \frac{9}{8x_n}$, $x_0 = \frac{1}{2}$ obtained from Newton-Raphson method. This series converges to :

(1) $\sqrt{2}$ (2) $\frac{2}{3}$ (3) $\frac{3}{2}$ (4) $\frac{8}{5}$

63. In solving the differential equation $y' = 2x, y(0) = 0$ using Euler's method, the interacts $y_n, n \in N$ satisfy :

(1) $y_n = 2x_n$ (2) $y_n = 2x_n - x_{n-1}$

(3) $y_n = x_n + x_{n-1}$ (4) $y_n = x_n x_{n-1}$

64. Hermite's interpolation formula is also called :

(1) osculating interpolation formula

(2) constant interpolation formula

(3) increasing interpolation formula

(4) decreasing interpolation formula

65. In Runge-Kutta fourth order method, for the initial value problem $\frac{dy}{dx}=f(x,y)$, $y(x_0)=y_0$ approximate value of y is given by $y_1=y_o+k$, where with usual notations :

(1) $k=\frac{1}{2}(k_1+2k_2+2k_3+k_4)$
(2) $k=\frac{1}{6}(k_1+2k_2+2k_3+k_4)$
(3) $k=\frac{1}{4}(k_1+k_2+k_3+k_4)$
(4) $k=\frac{1}{6}(k_1+3k_2+3k_3+k_4)$

66. The values of a, b, c respectively for which the formula $\int_a^b f(x)dx=h\left[af(0)+bf\left(\frac{h}{3}\right)+cf(h)\right]$ is exact for polynomials of as higher order as possible, are :

(1) $0, \frac{3}{4}, \frac{1}{4}$
(2) $0, \frac{1}{4}, \frac{3}{4}$
(3) $\frac{3}{4}, 0, \frac{1}{4}$
(4) $\frac{1}{4}, 0, \frac{3}{4}$

67. On what curve the function $I=\int_0^1\left[\left(\frac{dy}{dx}\right)^2+12xy\right]dx$ with $y(0) = 0$, $y(1) = 1$ can be extremized ?

(1) $y=x$
(2) $y=x^2$
(3) $y=x^3$
(4) $y=x^4$

68. Rayleigh-Ritz method is used to :

(1) find maxima
(2) find geodesics
(3) find minima
(4) solve boundary value problem

69. The simplification of the Euler-Lagrange equation is known as :

(1) Beltrami identity
(2) Liouveille's identity
(3) Hamilton identity
(4) Cauchy identity

70. The solution of the integral equation $\int_0^x \frac{y(t)}{x-t}dt=\sqrt{x}$ is :

(1) $y=1$
(2) $y=\frac{3}{2}$
(3) $y=\frac{1}{2}$
(4) $y=\frac{3}{4}$

71. Solution of Volterra integral equation $y(x)=1+x+\int_0^x (x-t)\,y(t)\,dt$, is :

(1) $y=e^x$
(2) $y=e^{-x}$
(3) $y=xe^x$
(4) $y=1+e^x$

72. Any non-trivial solution of the homogeneous integral equation for a certain value of λ is called :

(1) eigen value
(2) eigen function
(3) kernel
(4) resolvent kernel

73. The Lagrangian of a particle moving in a plane under the influence of a central potential is given by $L=\frac{1}{2}m\left(\dot{r}^2+r^2\dot{\theta}^2\right)-V(r)$. The generalized momenta corresponding to r and θ are given by :

(1) $m\dot{r}$ and $mr\dot{\theta}$
(2) $m\dot{r}^2$ and $mr\dot{\theta}$
(3) $m\dot{r}^2$ and $mr^2\dot{\theta}$
(4) $m\dot{r}$ and $mr^2\dot{\theta}$

74. If the generalized coordinate is angle θ, the corresponding generalized force has the dimensions of :

(1) velocity
(2) acceleration
(3) force
(4) displacement

75. The generalized displacement of a rigid body is a translation with rotation. This result is known as :

(1) Euler's theorem
(2) Law of inertia
(3) Chasle's theorem
(4) Law of force

76. Two lines of regressions are $X=-\frac{1}{18}Y+\lambda$ and $Y=-2X+\mu$; (λ, μ) being unknown and the mean of the distribution is $(-1, 2)$. Estimated value of X when $Y=10$ is :

(1) -2
(2) 2
(3) $-\frac{13}{9}$
(4) $\frac{13}{9}$

77. If partial correlation coefficient $r_{12.3} = 0$, then :

(1) $r_{12} = r_{13}r_{23}$ (2) $r_{23} = r_{21}r_{13}$

(3) $r_{31} = r_{12}r_{23}$ (4) $r_{12} = 1$

78. If $(X, Y) \sim B \vee N\,(0, 0, 1, 1, 0.8)$, then $1 + 2X + 3Y$ is distributed as :

(1) $N(0, 1)$ (2) $N(1, 13)$ (3) $N(1, 19)$ (4) $N(0, 19)$

79. In case of simple random sampling with replacement, the variance of the estimate of population mean is :

(1) $\frac{N-n}{nN}\sigma^2$ (2) $\frac{N-n}{nN} \cdot \frac{N-1}{N}\sigma^2$

(3) $\frac{N-n}{nN} \cdot \frac{N}{N-1}\sigma^2$ (4) $\frac{\sigma^2}{n}$

80. Which of the following statements is ***false*** ?

(1) Mean lies in between median and mode

(2) Mean, median and mode have the same unit

(3) In a moderately asymmetrical distribution, Mode = 3 Median – 2 Mean

(4) The median is not affected by the extreme values

81. The following LPP has the multiple optimal solutions :

Max. $Z = x + 3y$

Subject to

$2x + y \leq 10$

$x + 3y \leq 15$

$x, y \geq 0$

One of the points that gives optimal solution for the LPP is :

(1) $(5, 0)$ (2) $(2.7, 4.1)$ (3) $(9, 2)$ (4) $(2, 1)$

82. With 0.8 as the traffic intensity, the expected number of customers in M|M|1 system is :

(1) 4 (2) 3.2 (3) 5 (4) $\frac{20}{9}$

83. Number of observations saved in a 4×4 L. S. D. over a complete 3-way layout is :

(1) 4 (2) 12 (3) 24 (4) 48

84. A system is composed of three identical independent elements in series, each having the reliability 0.3, then reliability of the system is :

(1) 0.9 (2) 0.973 (3) 0.027 (4) 0.73

85. While solving an LPP by simplex method to find which variable to leave, ratio column is calculated. If all the ratios turn to be negative or undefined, it indicates the problem has :

(1) Degenerate solution (2) Unbounded solution

(3) No feasible solution (4) Multiple optimal solution

86. Row heading of a statistical table is known as :

(1) sub-title (2) stub (3) caption (4) reference note

87. Mean deviation is minimum when deviations are taken from :

(1) Mean (2) Median (3) Mode (4) Zero

88. A can hit target 2 times in 5 shots, B 3 times in 5 shots and C 4 times in 5 shots. They fire a volley (each try once to hit the target). The probability that two shots hit the target is :

(1) $\frac{24}{125}$ (2) $\frac{67}{125}$ (3) $\frac{121}{125}$ (4) $\frac{58}{125}$

89. The joint probability density function of a two-dimensional random variable (X, Y) is given by $f(x,y)=\begin{cases}2 & ; \; 0<x<1, 0<y<x \\ 0 & ; \; \text{elsewhere}\end{cases}$. The marginal density function of Y is :

(1) 2 (2) $2y$ (3) $2(1-y)$ (4) $2(y-1)$

90. For two random variable X and Y, the relation $E(XY) = E(X)\,E(Y)$ holds good :

(1) if X and Y are statistically independent

(2) for all X and Y

(3) if X and Y are identical

(4) if $X = Y$

91. A symmetric die is thrown 600 times. The lower bound for the probability of getting 80 to 120 sixes is :

(1) $\frac{19}{24}$ (2) $\frac{3}{4}$ (3) $\frac{1}{2}$ (4) $\frac{5}{24}$

92. The interval between two successive occurrence of a Poisson process $\{N(t),\ t \geq 0\}$ having parameter μ has :

(1) Poisson distribution with mean $\frac{1}{\mu}$

(2) Negative exponential distribution with mean $\frac{1}{\mu}$

(3) Negative exponential distribution with mean μ

(4) Poisson distribution with mean μ

93. For a binomial variate X, the mean is 6 and the standard deviation $\sqrt{2}$, then $P(X = 0)$ is :

(1) $\left(\frac{1}{3}\right)^6$ (2) $\left(\frac{2}{3}\right)^6$

(3) $\left(\frac{1}{3}\right)^9$ (4) $\left(\frac{2}{3}\right)^{18}$

94. Let $X_1, X_2, \ldots\ldots, X_n$ be n i.i.d. variates each with p.d.f. $f(x)$ and c. d. f. $F(x)$. The p.d.f. of the smallest order statistic is :

(1) $n[F(x)]^{n-1} \cdot f(x)$

(2) $1-[1-F(x)]^n$

(3) $[F(x)]^n$

(4) $n[1-F(x)]^{n-1} \cdot f(x)$

95. If for a Poisson variate X, $E(X^2)=6$, then $E(x)$ is :

(1) 6

(2) 3

(3) 2

(4) 1

96. In case of large sample single tailed test, the magnitude of the critical value of z at 5 percent level of significance is :

(1) 2.58

(2) 2.33

(3) 1.96

(4) 1.645

97. The hypothesis that the population variance has a specified value can be tested by :

(1) F-test

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(3) χ^2-test

(4) t-test

98. Which of the following is ***not*** true ?

(1) Probability of Type I error is also referred to as consumer's risk

(2) Every most powerful critical region is necessarily unbiased

(3) The standard deviation of the sampling distribution of a statistic is known as its standard error

(4) The value of test statistic which separates the critical region and acceptance region is called critical value

99. Which of the following assumptions is ***not*** associated with non-parametric tests ?

(1) Sample observations are independent

(2) Parent population from which the sample(s) have been drawn is normal

(3) The variate under study is continuous

(4) p. d. f. is continuous

100. Non-parametric test to be used for testing if two independent ordered samples differ in their central tendencies is :

(1) Sign test

(2) Run test

(3) Median test

(4) Mann-Whitney-Wilcoxon U-test

Professor & Head
Department of Mathematics
M.D. University, ROHTAK

(DO NOT OPEN THIS QUESTION BOOKLET BEFORE TIME OR UNTIL YOU ARE ASKED TO DO SO)

M.Phil./Ph.D./URS-EE-Oct.-2017

SUBJECT : Mathematics

B

10214

Sr. No. ....

Time : 1¼ **Hours** Max. Marks : **100** Total Questions : **100**

Roll No. (in figures)________________ (in words) ____________________________

Name ________________________ Father's Name________________________

Mother's Name____________________ Date of Examination____________________

(Signature of the Candidate) (Signature of the Invigilator)

SEAL

CANDIDATES MUST READ THE FOLLOWING INFORMATION/INSTRUCTIONS BEFORE STARTING THE QUESTION PAPER.

1. ***Candidates are required to attempt any 75 questions out of the given 100 multiple choice questions of 4/3 marks each. No credit will be given for more than 75 correct responses.***
2. The candidates ***must return*** the question booklet as well as OMR Answer-Sheet to the Invigilator concerned before leaving the Examination Hall, failing which a case of use of unfair-means/misbehaviour will be registered against him/her, in addition to lodging of an FIR with the police. Further the answer-sheet of such a candidate will not be evaluated.
3. In case there is any discrepancy in any question(s) in the Question Booklet, the same may be brought to the notice of the Controller of Examinations in writing **within two hours** after the test is over. No such complaint(s) will be entertained thereafter.
4. The candidate ***must not*** do any rough work or writing in the OMR Answer-Sheet. Rough work, if any, may be done in the question booklet itself. Answers ***must not*** be ticked in the question booklet.
5. ***There will be no negative marking. Each correct answer will be awarded 4/3 marks. Cutting, erasing, overwriting and more than one answer in OMR Answer-Sheet will be treated as incorrect answer.***
6. Use only **Black** or **Blue Ball Point Pen** of good quality in the OMR Answer-Sheet.
7. *Before answering the questions, the candidates should ensure that they have been supplied correct and complete booklet. Complaints, if any, regarding misprinting etc. will not be entertained* **30** *minutes after starting of the examination.*

1. A function $f(x)$ is a monotonic function if $f(x)$ is :
   (1) either increasing or decreasing function
   (2) only increasing function
   (3) only decreasing function
   (4) a constant function

2. The integral $\int_0^{\infty} \frac{\sin x}{x}\, dx$:
   (1) converges absolutely
   (2) does not converge
   (3) converges but not absolutely
   (4) does not exist

3. Let $f_n(x) = nxe^{-nx^2}, x \in [0,1]$, then which of the following is ***not*** a point sequence ?
   (1) $a_n = 2ne^{-2n}$
   (2) $a_n = ne^{-n}$
   (3) $a_n = 2ne^{-4n}$
   (4) $a_n = 0$

4. If $f(x,y) = \begin{cases} \frac{xy^2}{x^2+y^4} & , (x,y) \neq (0,0) \\ 0 & , (x,y) = (0,0) \end{cases}$, then the directional derivative along $u = (\sqrt{2}, \sqrt{2})$ at (0, 0) is :
   (1) $\sqrt{2}$
   (2) $\frac{1}{\sqrt{2}}$
   (3) $2\sqrt{2}$
   (4) $\frac{1}{2\sqrt{2}}$

5. If $f : X \to R$, $X \subseteq R^2$ and $(a, b) \in X$ is such that f_x, f_y are differentiable at (a, b), then $f_{xy}(a,b) = f_{yx}(a,b)$. This result is known as :
   (1) Schwarz's theorem
   (2) Young's theorem
   (3) Taylor's theorem
   (4) Implicit function theorem

6. The function $f(x,y) = x^4 + x^2y + y^2$ is :
   (1) minimum at (0, 0)
   (2) neither minimum nor maximum at (0, 0)
   (3) maximum at (0, 0)
   (4) discontinuous at (0, 0)

7. If $< E_i >$ is a sequence of Lebesgue measurable sets and m is the Lebesgue measure, then :

(1) $m(\cup E_i) = \sum mE_i$
(2) $m(\cup E_i) \geq \sum mE_i$
(3) $m(\cup E_i) \leq \sum mE_i$
(4) $m(\cup E_i) = 0$

8. If $\alpha > 0$ and $\beta > 0$ and $f(x) = \begin{cases} x^\alpha \sin \frac{1}{x^\beta} & , \ x \neq 0 \\ 0 & , \ x = 0 \end{cases}$, then $f(x)$ is of bounded variation in [0, 1] if :

(1) $\alpha < \beta$
(2) $1 + \alpha < \beta$
(3) $\alpha + \beta = 1$
(4) $\alpha > \beta$

9. Let $X = \{x : 0 < d(0, x) \leq 1$, and $x \in R^2\}$ where $0 = (0, 0)$ and d is the usual metric on X, then which of the following is ***not*** true ?

(1) X is closed
(2) X is bounded
(3) X is not compact
(4) X is compact

SEAL

10. If $X = [X_1 \ X_2 \ldots\ldots X_n]'$ is an n-tuple non-zero vector, then the $n \times n$ matrix $V = XX'$:

(1) has rank zero
(2) has rank 1
(3) has rank n
(4) is orthogonal

11. A symmetric die is thrown 600 times. The lower bound for the probability of getting 80 to 120 sixes is :

(1) $\frac{19}{24}$
(2) $\frac{3}{4}$
(3) $\frac{1}{2}$
(4) $\frac{5}{24}$

12. The interval between two successive occurrence of a Poisson process $\{N(t), t \geq 0$ having parameter μ has :

(1) Poisson distribution with mean $\frac{1}{\mu}$

(2) Negative exponential distribution with mean $\frac{1}{\mu}$

(3) Negative exponential distribution with mean μ

(4) Poisson distribution with mean μ

13. For a binomial variate X, the mean is 6 and the standard deviation $\sqrt{2}$, then $P(X = 0)$ is :

(1) $\left(\frac{1}{3}\right)^6$ (2) $\left(\frac{2}{3}\right)^6$

(3) $\left(\frac{1}{3}\right)^9$ (4) $\left(\frac{2}{3}\right)^{18}$

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(1) 6 (2) 3

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(1) Probability of Type I error is also referred to as consumer's risk

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19. Which of the following assumptions is ***not*** associated with non-parametric tests ?

(1) Sample observations are independent

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(4) p. d. f. is continuous

20. Non-parametric test to be used for testing if two independent ordered samples differ in their central tendencies is :

(1) Sign test (2) Run test

(3) Median test (4) Mann-Whitney-Wilcoxon U-test

21. Solution of Volterra integral equation $y(x) = 1 + x + \int_0^x (x-t)\, y(t)\, dt$, is :

(1) $y = e^x$ (2) $y = e^{-x}$ (3) $y = xe^x$ (4) $y = 1 + e^x$

22. Any non-trivial solution of the homogeneous integral equation for a certain value of λ is called :

(1) eigen value (2) eigen function (3) kernel (4) resolvent kernel

23. The Lagrangian of a particle moving in a plane under the influence of a central potential is given by $L = \frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\theta}^2\right) - V(r)$. The generalized momenta corresponding to r and θ are given by :

(1) $m\dot{r}$ and $mr\,\dot{\theta}$

(2) $m\dot{r}^2$ and $mr\,\dot{\theta}$

(3) $m\dot{r}^2$ and $mr^2\dot{\theta}$

(4) $m\dot{r}$ and $mr^2\dot{\theta}$

24. If the generalized coordinate is angle θ, the corresponding generalized force has the dimensions of :

(1) velocity

(2) acceleration

(3) force

(4) displacement

25. The generalized displacement of a rigid body is a translation with rotation. This result is known as :

(1) Euler's theorem

(2) Law of inertia

(3) Chasle's theorem

(4) Law of force

26. Two lines of regressions are $X = -\frac{1}{18}Y + \lambda$ and $Y = -2X + \mu$; (λ, μ) being unknown and the mean of the distribution is $(-1, 2)$. Estimated value of X when $Y = 10$ is :

(1) -2

(2) 2

(3) $-\frac{13}{9}$

(4) $\frac{13}{9}$

27. If partial correlation coefficient $r_{12.3} = 0$, then :

(1) $r_{12} = r_{13}r_{23}$

(2) $r_{23} = r_{21}r_{13}$

(3) $r_{31} = r_{12}r_{23}$

(4) $r_{12} = 1$

28. If $(X, Y) \sim B \vee N\ (0, 0, 1, 1, 0.8)$, then $1 + 2X + 3Y$ is distributed as :

(1) $N(0, 1)$

(2) $N(1, 13)$

(3) $N(1, 19)$

(4) $N(0, 19)$

29. In case of simple random sampling with replacement, the variance of the estimate of population mean is :

(1) $\frac{N-n}{nN}\sigma^2$

(2) $\frac{N-n}{nN}\cdot\frac{N-1}{N}\sigma^2$

(3) $\frac{N-n}{nN}\cdot\frac{N}{N-1}\sigma^2$

(4) $\frac{\sigma^2}{n}$

30. Which of the following statements is ***false*** ?

(1) Mean lies in between median and mode

(2) Mean, median and mode have the same unit

(3) In a moderately asymmetrical distribution, Mode = 3 Median – 2 Mean

(4) The median is not affected by the extreme values

31. Solution of $\frac{dy}{dx}=\frac{y}{x}+x\tan\frac{y}{x}$ is :

(1) $\log\cot\frac{y}{x}=x+c$

(2) $\log\tan\frac{y}{x}=x+c$

(3) $\log\cot\frac{y}{2x}=x+c$

(4) $\log\tan\frac{y}{2x}=x+c$

32. Solution of $y=xp-p^2$ is :

(1) $y=\log x+c$ (2) $y=cx-c^2$ (3) $y=3x-c$ (4) $y=cx-cx^2$

33. P. I. of $(D^2-6D+9)y=8e^{3x}$ is :

(1) $4xe^{3x}$ (2) $\frac{x}{2}e^{3x}$ (3) $2x^2e^{3x}$ (4) $4x^2e^{3x}$

34. Solving $y''-2y'+y=e^x\log x$ by variation for parameters, the value of Wronskion is :

(1) e^{-2x} (2) e^{2x} (3) xe^{2x} (4) $x\bar{e}^{2x}$

35. Green's function of the boundary value problem $y''=0, y(0)=y(1)=0$ is given by :

(1) $G(x,t)=x(1-t); 0 \le x < t$

(2) $G(x,t)=xt; t < x \le 1$

(3) $G(x,t)=x^2(1-t); 0 \le x < t$

(4) $G(x,t)=x(1-t^2); t < x \le 1$

36. The general solution of $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ is of the form :

(1) $u=cf(x-iy)$

(2) $u=cf(x+y)$

(3) $u=f(x+y)+g(x-y)$

(4) $u=f(x+iy)+g(x-iy)$

37. The initial value problem $u_x+u_y=1, u(s,s)=\sin s, 0 \le s \le 1$ has :

(1) no solution

(2) a unique solution

(3) two solutions

(4) infinitely many solutions

38. The region in which the differential equation $yu_{xx}+2xyu_{xy}+xu_{yy}=u_x+u_y$, is hyperbolic, is :

(1) $xy>0$

(2) $xy \ne 0$

(3) $xy>1$

(4) $xy \ne 1$

39. The partial differential equation formed by eliminating arbitrary function from the equation $z=f(x^2-y^2)$ is :

(1) $xp+yq=0$

(2) $xq+yp=0$

(3) $\frac{x}{y}=q$

(4) $\frac{x}{y}=p$

40. P. I. of $(2D^2-3DD'+D'^2)z=e^{x+2y}$ is :

(1) x^2e^{x+2y}

(2) xe^{x+2y}

(3) $-\frac{x}{3}e^{x+2y}$

(4) $-\frac{x}{2}e^{x+2y}$

41. The function $f(z)=\bar{z}$ is :

(1) Everywhere differentiable

(2) Nowhere differentiable

(3) Differentiable only at $z=0$

(4) Differentiable everywhere except at $z=0$

42. $\int_{r} \frac{1}{z}\, dz$, where $r(t) = \sin t + i \cos t,\ 0 \le t \le 2\pi$, is :

(1) -2π (2) $4\pi i$ (3) $2\pi i$ (4) $-2\pi i$

43. Using Cauchy's integral formula for derivatives $\int_{C} \frac{\sin z}{z^4}\, dz$, where C is the circle $|z| = 2$, is :

(1) $\pi i/2$ (2) $\pi i/3$ (3) $-\pi i/3$ (4) $-\pi i/2$

44. For the function $f(z) = z^3 \sin\left(\frac{1}{z}\right)$, the point $z = 0$ is :

(1) zero of order one (2) pole of order one
(3) essential singularity (4) zero of order two

45. Residue of $f(z) = e^{3/z}$ at $z = 0$, is :

(1) $\frac{1}{3}$ (2) 3 (3) e^3 (4) ∞

46. Using Cauchy Residue theorem, $\oint_{C} \frac{2z+6}{z^2+4} dz$, where C is $|z - i| = 2$, is :

(1) $\frac{\pi}{2}(2 + 3i)$ (2) $\frac{\pi}{2}(3 + 2i)$ (3) $\pi(2 + 3i)$ (4) $\pi(3 + 2i)$

47. The mapping $f(z) = ze^{z^2-2}$ is not conformal at $z =$

(1) $\pm \frac{i}{\sqrt{2}}$ (2) $\pm \frac{1}{\sqrt{2}}$ (3) $\pm \sqrt{2} i$ (4) 0

48. If 20 dictionaries in a library contain 41727 pages, then one of the dictionaries must have at least :

(1) 2084 pages (2) 2085 pages (3) 2086 pages (4) 2087 pages

49. In how many ways can a person can invite his 5 friends for dinner ?

(1) 31 (2) 32 (3) 33 (4) 120

50. For any integer $n > 2$, which of the following is true for the Euler's function $\phi(n)$?

(1) $\phi(n)$ is zero
(2) $\phi(n)$ is even
(3) $\phi(n)$ is odd
(4) $\phi(n)$ is rational number

51. For the matrix $A = \begin{bmatrix} 2 & -2 & 3 \\ -2 & -1 & 6 \\ 1 & 2 & 0 \end{bmatrix}$, one of the eigen values is 3, the other two eigen values are :

(1) 2, –5 (2) 3, 5 (3) 3, –5 (4) 2, 5

52. Let V be the vector space of ordered pairs of complex numbers over the real field R, then the dimension of V is :

(1) 6 (2) 4 (3) 2 (4) 1

53. Let T be a linear transformation on a vector space V such that $T^2 - T + 1 = 0$, then T is :

(1) singular (2) invertible (3) not invertible (4) idempotent

54. A real quadratic form $X^T AX$ in three variables is equivalent to the diagonal form $3y_1^2 - 4y_2^2 + 5y_3^2$. Then, the quadratic form $X^T AX$ is :

(1) positive definite
(2) negative definite
(3) positive semi-definite
(4) indefinite

55. The orthogonal complement of inner product space V is :

(1) zero subspace
(2) any subspace
(3) V itself
(4) None of these

56. If $z = -\sqrt{3} - i$, then a value of z^4 is :

(1) $8(1 + i\sqrt{3})$ (2) $8(1 - i\sqrt{3})$ (3) $8(-1 + i\sqrt{3})$ (4) $8(\sqrt{3} - i)$

57. If $w = u + iv$, $z = x + iy$, then the image of the line $x = -3$ under the complex mapping $w = z^2$ is :

(1) $u = -3v$ (2) $u = -3 + \frac{v^2}{4}$ (3) $v = 9 - \frac{u^2}{36}$ (4) $u = 9 - \frac{v^2}{36}$

58. Value of $i^{1/3}$ is :

(1) $\frac{\sqrt{3}}{2}-\frac{1}{2}i$ (2) $\frac{\sqrt{3}}{2}+\frac{1}{2}i$ (3) $\frac{1}{2}+\frac{\sqrt{3}}{2}i$ (4) $\frac{1}{2}+\frac{1}{2}i$

59. Solution of $e^{z-1}=-ie^3$ is :

(1) $4+(2n-1)\pi i$

(2) $4+\frac{1}{2}(2n-1)\pi i$

(3) $4+\frac{1}{2}(4n-1)\pi i$

(4) $\frac{1}{4}(2n-1)\pi i$

60. Solution of the equation $\cos z = i \sin z$ is :

(1) $z=\frac{2(n+1)}{3}\pi i$

(2) $z=(4n+1)\frac{\pi i}{2}$

(3) $\left(\frac{2n+1}{2}\right)\pi i$

(4) No solution

61. The generators of the group $G=\{a, a^2, a^3, a^4, a^5, a^6=e\}$ are :

(1) a and a^5 (2) a^3 and a^5 (3) a^2 and a^3 (4) a and a^3

62. If N is the set of natural numbers, then under the binary operation $a \,.\, b = a + b$,$(N, .)$ is :

(1) group (2) semi-group (3) quasi-group (4) monoid

63. If $C = (1, 2, 3, 4)$, then C^2 is :

(1) (1, 4) (2, 3) (2) (1, 3) (3) (1, 3) (2, 4) (4) (2, 4)

64. Which of the following is *not* true ?

The relation of isomorphism in the set of all groups :

(1) satisfies reflexivity

(2) satisfies anti-symmetry

(3) satisfies transitivity

(4) is an equivalence relation

65. $[Q(\sqrt{2}, \sqrt{3}) : Q] =$

(1) 4 (2) 3 (3) 2 (4) 1

66. Which of the following is ***not*** a field ?

(1) $\frac{z}{2z}$ (2) $\frac{z}{3z}$ (3) $\frac{z}{4z}$ (4) $\frac{z}{5z}$

67. Given a field F and the set M of all 2×2 matrices of the form $\begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix}$ for $a, b \in F$, then which of the following is ***not*** true ?

(1) M is left ideal in F

(2) M is right ideal in F

(3) M is subring of F

(4) M is right ideal but not left ideal in F

68. Which of the following is ***not*** true ?

(1) Every metric space is first countable

(2) A metric space is second countable iff it is separable

(3) Every metric space is second countable

(4) Any open subspace of a separable space is separable

69. Which of the following is ***not*** correct ?

(1) Every compact metric space is complete

(2) A compact Hausdorff space is normal

(3) A compact subspace of a Hausdorff space is closed

(4) Every metric space is compact Hausdorff space

70. If $X = \{a, b, c\}$, $T = \{\phi, X, \{a, c\}, \{b\}\}$, then the topological space (X, T) is :

(1) not a connected space

(2) A connected space

(3) not a Hausdorff space

(4) not a compact space

71. $\left(\frac{\Delta^2}{E}\right)e^x \cdot \frac{Ee^x}{\Delta^2 e^x} =$

(1) e^x (2) xe^x (3) $(x+1)e^x$ (4) $\frac{e^x}{x}$

72. Consider the series $x_{n+1} = \frac{x_n}{2} + \frac{9}{8x_n}, x_0 = \frac{1}{2}$ obtained from Newton-Raphson method. This series converges to :

(1) $\sqrt{2}$ (2) $\frac{2}{3}$ (3) $\frac{3}{2}$ (4) $\frac{8}{5}$

73. In solving the differential equation $y' = 2x, y(0) = 0$ using Euler's method, the interacts $y_n, n \in N$ satisfy :

(1) $y_n = 2x_n$

(2) $y_n = 2x_n - x_{n-1}$

(3) $y_n = x_n + x_{n-1}$

(4) $y_n = x_n x_{n-1}$

74. Hermite's interpolation formula is also called :

(1) osculating interpolation formula

(2) constant interpolation formula

(3) increasing interpolation formula

(4) decreasing interpolation formula

75. In Runge-Kutta fourth order method, for the initial value problem $\frac{dy}{dx} = f(x,y)$, $y(x_0) = y_0$ approximate value of y is given by $y_1 = y_o + k$, where with usual notations :

(1) $k = \frac{1}{2}(k_1 + 2k_2 + 2k_3 + k_4)$

(2) $k = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

(3) $k = \frac{1}{4}(k_1 + k_2 + k_3 + k_4)$

(4) $k = \frac{1}{6}(k_1 + 3k_2 + 3k_3 + k_4)$

76. The values of a, b, c respectively for which the formula $\int_a^b f(x)dx = h\left[af(0) + bf\left(\frac{h}{3}\right) + cf(h)\right]$ is exact for polynomials of as higher order as possible, are :

(1) $0, \frac{3}{4}, \frac{1}{4}$ (2) $0, \frac{1}{4}, \frac{3}{4}$ (3) $\frac{3}{4}, 0, \frac{1}{4}$ (4) $\frac{1}{4}, 0, \frac{3}{4}$

77. On what curve the function $I = \int_0^1 \left[\left(\frac{dy}{dx} \right)^2 + 12xy \right] dx$ with $y(0) = 0$, $y(1) = 1$ can be extremized ?

(1) $y = x$ (2) $y = x^2$ (3) $y = x^3$ (4) $y = x^4$

78. Rayleigh-Ritz method is used to :

(1) find maxima
(2) find geodesics
(3) find minima
(4) solve boundary value problem

79. The simplification of the Euler-Lagrange equation is known as :

(1) Beltrami identity
(2) Liouveille's identity
(3) Hamilton identity
(4) Cauchy identity

80. The solution of the integral equation $\int_0^x \frac{y(t)}{x-t} dt = \sqrt{x}$ is :

(1) $y = 1$ (2) $y = \frac{3}{2}$ (3) $y = \frac{1}{2}$ (4) $y = \frac{3}{4}$

81. If $n(A) = 115$, $n(B) = 225$, $n(A - B) = 73$, then $n(A \cup B) =$

(1) 265 (2) 278 (3) 295 (4) 298

82. The set of interior points of which of the following sets is ***not*** empty ?

(1) R (2) N (3) Z (4) I

83. If X is the set of even natural numbers less than 8 and Y is the set of odd prime numbers less than equal to 7, then the number of relations from X to Y is :

(1) 9 (2) $2^9 - 1$ (3) 2^{9-1} (4) 2^9

84. The sequence $S_n = \begin{cases} 2 & ; \text{ when } n \text{ is even} \\ \text{lowest prime factor } (\neq 1) \text{ of } n & , \text{ when } n \text{ is odd} \end{cases}$ has limit point :

(1) 2
(2) countable in number
(3) 1, 2, 3, 4, ...
(4) uncountable in number

85. The series $\sum_{n=1}^{\infty} \frac{1}{\left(1+\frac{1}{n}\right)^{n^2}}$ is :

(1) convergent

(2) conditionally convergent

(3) divergent

(4) oscillatory

86. $\lim_{n\to\infty}\left[\frac{n}{n^2}+\frac{n}{(n^2+1^2)}+\frac{n}{(n^2+2^2)}+\ldots\ldots\right]=$

(1) 0

(2) $\frac{\pi}{3}$

(3) $\frac{\pi}{4}$

(4) $\frac{-\pi}{4}$

87. $\lim_{x\to 0}\frac{\sinh x-\sin x}{x\sin^2 x}=$

(1) $\frac{1}{6}$

(2) $\frac{1}{4}$

(3) $\frac{1}{3}$

(4) $\frac{1}{2}$

88. Let $f(x)=\begin{cases}\frac{e^{1/x}}{1+e^{1/x}} & , \quad x\neq 0\\ 0 & , \quad x=0\end{cases}$, then which of the following is true ?

(1) $f(x)$ is continuous at $x = 0$

(2) $f(x)$ is discontinuous at $x = 0$

(3) $f(x)$ is differentiable at $x = 0$

(4) $f(x)$ has discontinuity of first kind from left at $x = 0$

89. If $f(x + y) = f(x)\, f(y)\ \forall\ x, y$ and $f(5) = -2$ and $f'(0) = 3$, then the value of $f'(5) =$

(1) –3

(2) –5

(3) –6

(4) 6

90. What is the abscissa of the point at which the tangent to the curve $y = x\,(x - 1)$ is parallel to the chord joining the extrimities of the curve in the interval [1, 2] ?

(1) 5/4

(2) 5/3

(3) 4/3

(4) 3/2

91. The following LPP has the multiple optimal solutions :

Max. $Z = x + 3y$

Subject to

$2x + y \leq 10$

$x + 3y \leq 15$

$x, y \geq 0$

One of the points that gives optimal solution for the LPP is :

(1) (5, 0) (2) (2.7, 4.1) (3) (9, 2) (4) (2, 1)

92. With 0.8 as the traffic intensity, the expected number of customers in M | M | 1 system is :

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93. Number of observations saved in a 4×4 L. S. D. over a complete 3-way layout is :

(1) 4 (2) 12 (3) 24 (4) 48

94. A system is composed of three identical independent elements in series, each having the reliability 0.3, then reliability of the system is :

(1) 0.9 (2) 0.973 (3) 0.027 (4) 0.73

95. While solving an LPP by simplex method to find which variable to leave, ratio column is calculated. If all the ratios turn to be negative or undefined, it indicates the problem has :

(1) Degenerate solution (2) Unbounded solution

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96. Row heading of a statistical table is known as :

(1) sub-title (2) stub (3) caption (4) reference note

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(1) Mean (2) Median (3) Mode (4) Zero

98. A can hit target 2 times in 5 shots, B 3 times in 5 shots and C 4 times in 5 shots. They fire a volley (each try once to hit the target). The probability that two shots hit the target is :

(1) $\frac{24}{125}$ (2) $\frac{67}{125}$ (3) $\frac{121}{125}$ (4) $\frac{58}{125}$

99. The joint probability density function of a two-dimensional random variable (X, Y) is given by $f(x,y) = \begin{cases} 2 & ; \quad 0 < x < 1, 0 < y < x \\ 0 & ; \quad \text{elsewhere} \end{cases}$. The marginal density function of Y is :

(1) 2 (2) $2y$ (3) $2(1-y)$ (4) $2(y-1)$

100. For two random variable X and Y, the relation $E(XY) = E(X)\,E(Y)$ holds good :

(1) if X and Y are statistically independent

(2) for all X and Y

(3) if X and Y are identical

(4) if $X = Y$

Professor & Head
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(DO NOT OPEN THIS QUESTION BOOKLET BEFORE TIME OR UNTIL YOU ARE ASKED TO DO SO)

M.Phil./Ph.D./URS-EE-Oct.-2017

SUBJECT : Mathematics

C

Sr. No. 10215

Time : **1¼ Hours** Max. Marks : **100** Total Questions : **100**

Roll No. (in figures)__________ (in words) __________

Name __________ Father's Name __________

Mother's Name __________ Date of Examination __________

__________ __________

(Signature of the Candidate) (Signature of the Invigilator)

SEAL

CANDIDATES MUST READ THE FOLLOWING INFORMATION/INSTRUCTIONS BEFORE STARTING THE QUESTION PAPER.

1. ***Candidates are required to attempt any 75 questions out of the given 100 multiple choice questions of 4/3 marks each. No credit will be given for more than 75 correct responses.***
2. The candidates ***must return*** the question booklet as well as OMR Answer-Sheet to the Invigilator concerned before leaving the Examination Hall, failing which a case of use of unfair-means/misbehaviour will be registered against him/her, in addition to lodging of an FIR with the police. Further the answer-sheet of such a candidate will not be evaluated.
3. In case there is any discrepancy in any question(s) in the Question Booklet, the same may be brought to the notice of the Controller of Examinations in writing **within two hours** after the test is over. No such complaint(s) will be entertained thereafter.
4. The candidate ***must not*** do any rough work or writing in the OMR Answer-Sheet. Rough work, if any, may be done in the question booklet itself. Answers ***must not*** be ticked in the question booklet.
5. ***There will be no negative marking. Each correct answer will be awarded 4/3 marks. Cutting, erasing, overwriting and more than one answer in OMR Answer-Sheet will be treated as incorrect answer.***
6. Use only **Black** or **Blue Ball Point Pen** of good quality in the OMR Answer-Sheet.
7. *Before answering the questions, the candidates should ensure that they have been supplied correct and complete booklet. Complaints, if any, regarding misprinting etc. will not be entertained* **30** *minutes after starting of the examination.*

C

1. The generators of the group $G = \{a, a^2, a^3, a^4, a^5, a^6 = e\}$ are :

   (1) a and a^5 (2) a^3 and a^5 (3) a^2 and a^3 (4) a and a^3

2. If N is the set of natural numbers, then under the binary operation $a \,.\, b = a + b, (N, .)$ is :

   (1) group (2) semi-group (3) quasi-group (4) monoid

3. If $C = (1, 2, 3, 4)$, then C^2 is :

   (1) (1, 4) (2, 3) (2) (1, 3) (3) (1, 3) (2, 4) (4) (2, 4)

4. Which of the following is ***not*** true ?

   The relation of isomorphism in the set of all groups :

   (1) satisfies reflexivity (2) satisfies anti-symmetry

   (3) satisfies transitivity (4) is an equivalence relation

5. $\left[Q\left(\sqrt{2}, \sqrt{3}\right) : Q\right] =$

   (1) 4 (2) 3 (3) 2 (4) 1

6. Which of the following is ***not*** a field ?

   (1) $\frac{z}{2z}$ (2) $\frac{z}{3z}$ (3) $\frac{z}{4z}$ (4) $\frac{z}{5z}$

7. Given a field F and the set M of all 2×2 matrices of the form $\begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix}$ for $a, b \in F$, then which of the following is ***not*** true ?

   (1) M is left ideal in F (2) M is right ideal in F

   (3) M is subring of F (4) M is right ideal but not left ideal in F

8. Which of the following is ***not*** true ?

   (1) Every metric space is first countable

   (2) A metric space is second countable iff it is separable

   (3) Every metric space is second countable

   (4) Any open subspace of a separable space is separable

9. Which of the following is ***not*** correct ?

(1) Every compact metric space is complete

(2) A compact Hausdorff space is normal

(3) A compact subspace of a Hausdorff space is closed

(4) Every metric space is compact Hausdorff space

10. If $X = \{a, b, c\}$, $T = \{\phi, X, \{a, c\}, \{b\}\}$, then the topological space (X, T) is :

(1) not a connected space (2) A connected space

(3) not a Hausdorff space (4) not a compact space

11. For the matrix $A = \begin{bmatrix} 2 & -2 & 3 \\ -2 & -1 & 6 \\ 1 & 2 & 0 \end{bmatrix}$, one of the eigen values is 3, the other two eigen values are :

(1) 2, –5 (2) 3, 5 (3) 3, –5 (4) 2, 5

12. Let V be the vector space of ordered pairs of complex numbers over the real field R, then the dimension of V is :

(1) 6 (2) 4 (3) 2 (4) 1

13. Let T be a linear transformation on a vector space V such that $T^2 - T + 1 = 0$, then T is :

(1) singular (2) invertible (3) not invertible (4) idempotent

14. A real quadratic form $X^T AX$ in three variables is equivalent to the diagonal form $3y_1^2 - 4y_2^2 + 5y_3^2$. Then, the quadratic form $X^T AX$ is :

(1) positive definite (2) negative definite

(3) positive semi-definite (4) indefinite

15. The orthogonal complement of inner product space V is :

(1) zero subspace (2) any subspace

(3) V itself (4) None of these

16. If $z=-\sqrt{3}-i$, then a value of z^4 is :

(1) $8(1+i\sqrt{3})$ (2) $8(1-i\sqrt{3})$ (3) $8(-1+i\sqrt{3})$ (4) $8(\sqrt{3}-i)$

17. If $w = u + iv$, $z = x+ iy$, then the image of the line $x = -3$ under the complex mapping $w = z^2$ is :

(1) $u=-3v$ (2) $u=-3+\frac{v^2}{4}$ (3) $v=9-\frac{u^2}{36}$ (4) $u=9-\frac{v^2}{36}$

18. Value of $i^{1/3}$ is :

(1) $\frac{\sqrt{3}}{2}-\frac{1}{2}i$ (2) $\frac{\sqrt{3}}{2}+\frac{1}{2}i$ (3) $\frac{1}{2}+\frac{\sqrt{3}}{2}i$ (4) $\frac{1}{2}+\frac{1}{2}i$

19. Solution of $e^{z-1}=-ie^3$ is :

(1) $4+(2n-1)\pi i$

(2) $4+\frac{1}{2}(2n-1)\pi i$

(3) $4+\frac{1}{2}(4n-1)\pi i$

(4) $\frac{1}{4}(2n-1)\pi i$

20. Solution of the equation $\cos z = i \sin z$ is :

(1) $z=\frac{2(n+1)}{3}\pi i$

(2) $z=(4n+1)\frac{\pi i}{2}$

(3) $\left(\frac{2n+1}{2}\right)\pi i$

(4) No solution

21. If $n(A) = 115$, $n(B) = 225$, $n(A - B) = 73$, then $n(A\cup B) =$

(1) 265 (2) 278 (3) 295 (4) 298

22. The set of interior points of which of the following sets is ***not*** empty ?

(1) R (2) N (3) Z (4) I

23. If X is the set of even natural numbers less than 8 and Y is the set of odd prime numbers less than equal to 7, then the number of relations from X to Y is :

(1) 9 (2) 2^9-1 (3) 2^{9-1} (4) 2^9

24. The sequence $S_n=\begin{cases} 2 & \text{, when } n \text{ is even} \\ \text{lowest prime factor } (\neq 1) \text{ of } n & \text{, when } n \text{ is odd} \end{cases}$ has limit point :

(1) 2

(2) countable in number

(3) 1, 2, 3, 4, ...

(4) uncountable in number

25. The series $\sum_{n=1}^{\infty} \frac{1}{\left(1+\frac{1}{n}\right)^{n^2}}$ is :

(1) convergent (2) conditionally convergent

(3) divergent (4) oscillatory

26. $\lim_{n\to\infty}\left[\frac{n}{n^2}+\frac{n}{(n^2+1^2)}+\frac{n}{(n^2+2^2)}+......\right]=$

(1) 0 (2) $\frac{\pi}{3}$ (3) $\frac{\pi}{4}$ (4) $\frac{-\pi}{4}$

27. $\lim_{x\to 0}\frac{\sinh x-\sin x}{x\sin^2 x}=$

(1) $\frac{1}{6}$ (2) $\frac{1}{4}$ (3) $\frac{1}{3}$ (4) $\frac{1}{2}$

28. Let $f(x)=\begin{cases}\frac{e^{1/x}}{1+e^{1/x}} & , x\neq 0\\ 0 & , x=0\end{cases}$, then which of the following is true ?

(1) $f(x)$ is continuous at $x = 0$

(2) $f(x)$ is discontinuous at $x = 0$

(3) $f(x)$ is differentiable at $x = 0$

(4) $f(x)$ has discontinuity of first kind from left at $x = 0$

29. If $f(x + y) = f(x) f(y)\ \forall\ x, y$ and $f(5) = -2$ and $f'(0)=3$, then the value of $f'(5)=$

(1) −3 (2) −5 (3) −6 (4) 6

30. What is the abscissa of the point at which the tangent to the curve $y = x\ (x - 1)$ is parallel to the chord joining the extrimities of the curve in the interval [1, 2] ?

(1) 5/4 (2) 5/3 (3) 4/3 (4) 3/2

31. A symmetric die is thrown 600 times. The lower bound for the probability of getting 80 to 120 sixes is :

(1) $\frac{19}{24}$ (2) $\frac{3}{4}$ (3) $\frac{1}{2}$ (4) $\frac{5}{24}$

32. The interval between two successive occurrence of a Poisson process $\{N(t),\ t \geq 0\}$ having parameter μ has :

(1) Poisson distribution with mean $\frac{1}{\mu}$

(2) Negative exponential distribution with mean $\frac{1}{\mu}$

(3) Negative exponential distribution with mean μ

(4) Poisson distribution with mean μ

33. For a binomial variate X, the mean is 6 and the standard deviation $\sqrt{2}$, then $P(X = 0)$ is :

(1) $\left(\frac{1}{3}\right)^6$ (2) $\left(\frac{2}{3}\right)^6$

(3) $\left(\frac{1}{3}\right)^9$ (4) $\left(\frac{2}{3}\right)^{18}$

34. Let $X_1, X_2, \ldots\ldots, X_n$ be n i.i.d. variates each with p.d.f. $f(x)$ and c. d. f. $F(x)$. The p.d.f. of the smallest order statistic is :

(1) $n[F(x)]^{n-1} \cdot f(x)$ (2) $1-[1-F(x)]^n$

(3) $[F(x)]^n$ (4) $n[1-F(x)]^{n-1} \cdot f(x)$

35. If for a Poisson variate X, $E(X^2) = 6$, then $E(x)$ is :

(1) 6 (2) 3

(3) 2 (4) 1

36. In case of large sample single tailed test, the magnitude of the critical value of z at 5 percent level of significance is :

(1) 2.58 (2) 2.33

(3) 1.96 (4) 1.645

37. The hypothesis that the population variance has a specified value can be tested by :

(1) F-test (2) Z-test

(3) χ^2-test (4) t-test

38. Which of the following is ***not*** true ?

(1) Probability of Type I error is also referred to as consumer's risk

(2) Every most powerful critical region is necessarily unbiased

(3) The standard deviation of the sampling distribution of a statistic is known as its standard error

(4) The value of test statistic which separates the critical region and acceptance region is called critical value

39. Which of the following assumptions is ***not*** associated with non-parametric tests ?

(1) Sample observations are independent

(2) Parent population from which the sample(s) have been drawn is normal

(3) The variate under study is continuous

(4) p. d. f. is continuous

40. Non-parametric test to be used for testing if two independent ordered samples differ in their central tendencies is :

(1) Sign test (2) Run test

(3) Median test (4) Mann-Whitney-Wilcoxon U-test

41. $\left(\frac{\Delta^2}{E}\right)e^x \cdot \frac{Ee^x}{\Delta^2 e^x} =$

(1) e^x (2) xe^x (3) $(x+1)e^x$ (4) $\frac{e^x}{x}$

42. Consider the series $x_{n+1} = \frac{x_n}{2} + \frac{9}{8x_n}, x_0 = \frac{1}{2}$ obtained from Newton-Raphson method. This series converges to :

(1) $\sqrt{2}$ (2) $\frac{2}{3}$ (3) $\frac{3}{2}$ (4) $\frac{8}{5}$

43. In solving the differential equation $y' = 2x, y(0) = 0$ using Euler's method, the interacts $y_n, n \in N$ satisfy :

(1) $y_n = 2x_n$ (2) $y_n = 2x_n - x_{n-1}$

(3) $y_n = x_n + x_{n-1}$ (4) $y_n = x_n x_{n-1}$

44. Hermite's interpolation formula is also called :

(1) osculating interpolation formula

(2) constant interpolation formula

(3) increasing interpolation formula

(4) decreasing interpolation formula

45. In Runge-Kutta fourth order method, for the initial value problem $\frac{dy}{dx} = f(x,y)$, $y(x_0) = y_0$ approximate value of y is given by $y_1 = y_o + k$, where with usual notations :

(1) $k = \frac{1}{2}(k_1 + 2k_2 + 2k_3 + k_4)$ (2) $k = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

(3) $k = \frac{1}{4}(k_1 + k_2 + k_3 + k_4)$ (4) $k = \frac{1}{6}(k_1 + 3k_2 + 3k_3 + k_4)$

46. The values of a, b, c respectively for which the formula $\int_a^b f(x)dx = h\left[af(0)+bf\left(\frac{h}{3}\right)+cf(h)\right]$ is exact for polynomials of as higher order as possible, are :

(1) $0, \frac{3}{4}, \frac{1}{4}$ (2) $0, \frac{1}{4}, \frac{3}{4}$ (3) $\frac{3}{4}, 0, \frac{1}{4}$ (4) $\frac{1}{4}, 0, \frac{3}{4}$

47. On what curve the function $I = \int_0^1\left[\left(\frac{dy}{dx}\right)^2 + 12xy\right]dx$ with $y(0) = 0$, $y(1) = 1$ can be extremized ?

(1) $y = x$ (2) $y = x^2$ (3) $y = x^3$ (4) $y = x^4$

48. Rayleigh-Ritz method is used to :

(1) find maxima (2) find geodesics

(3) find minima (4) solve boundary value problem

49. The simplification of the Euler-Lagrange equation is known as :

(1) Beltrami identity (2) Liouveille's identity

(3) Hamilton identity (4) Cauchy identity

50. The solution of the integral equation $\int_0^x \frac{y(t)}{x-t}dt = \sqrt{x}$ is :

(1) $y = 1$ (2) $y = \frac{3}{2}$ (3) $y = \frac{1}{2}$ (4) $y = \frac{3}{4}$

51. The function $f(z) = \bar{z}$ is :

(1) Everywhere differentiable

(2) Nowhere differentiable

(3) Differentiable only at $z = 0$

(4) Differentiable everywhere except at $z = 0$

52. $\int_r \frac{1}{z}\,dz$, where $r(t) = \sin t + i \cos t, 0 \le t \le 2\pi$, is :

(1) -2π (2) $4\pi i$ (3) $2\pi i$ (4) $-2\pi i$

53. Using Cauchy's integral formula for derivatives $\int_C \frac{\sin z}{z^4}\,dz$, where C is the circle $|z| = 2$, is :

(1) $\pi i/2$ (2) $\pi i/3$ (3) $-\pi i/3$ (4) $-\pi i/2$

54. For the function $f(z) = z^3 \sin\left(\frac{1}{z}\right)$, the point $z = 0$ is :

(1) zero of order one (2) pole of order one

(3) essential singularity (4) zero of order two

55. Residue of $f(z) = e^{3/z}$ at $z = 0$, is :

(1) $\frac{1}{3}$ (2) 3 (3) e^3 (4) ∞

56. Using Cauchy Residue theorem, $\oint_C \frac{2z+6}{z^2+4}\,dz$, where C is $|z - i| = 2$, is :

(1) $\frac{\pi}{2}(2+3i)$ (2) $\frac{\pi}{2}(3+2i)$ (3) $\pi(2+3i)$ (4) $\pi(3+2i)$

57. The mapping $f(z) = ze^{z^2-2}$ is not conformal at $z =$

(1) $\pm\frac{i}{\sqrt{2}}$ (2) $\pm\frac{1}{\sqrt{2}}$ (3) $\pm\sqrt{2}i$ (4) 0

58. If 20 dictionaries in a library contain 41727 pages, then one of the dictionaries must have at least :

(1) 2084 pages (2) 2085 pages (3) 2086 pages (4) 2087 pages

59. In how many ways can a person can invite his 5 friends for dinner ?

(1) 31 (2) 32 (3) 33 (4) 120

60. For any integer $n > 2$, which of the following is true for the Euler's function $\phi(n)$?

(1) $\phi(n)$ is zero
(2) $\phi(n)$ is even
(3) $\phi(n)$ is odd
(4) $\phi(n)$ is rational number

61. Solution of Volterra integral equation $y(x) = 1 + x + \int_0^x (x-t)\, y(t)\, dt$, is :

(1) $y = e^x$
(2) $y = e^{-x}$
(3) $y = xe^x$
(4) $y = 1 + e^x$

62. Any non-trivial solution of the homogeneous integral equation for a certain value of λ is called :

(1) eigen value
(2) eigen function
(3) kernel
(4) resolvent kernel

63. The Lagrangian of a particle moving in a plane under the influence of a central potential is given by $L = \frac{1}{2} m\left(\dot{r}^2 + r^2\dot{\theta}^2\right) - V(r)$. The generalized momenta corresponding to r and θ are given by :

(1) $m\dot{r}$ and $mr\,\dot{\theta}$
(2) $m\dot{r}^2$ and $mr\,\dot{\theta}$
(3) $m\dot{r}^2$ and $mr^2\dot{\theta}$
(4) $m\dot{r}$ and $mr^2\dot{\theta}$

64. If the generalized coordinate is angle θ, the corresponding generalized force has the dimensions of :

(1) velocity
(2) acceleration
(3) force
(4) displacement

65. The generalized displacement of a rigid body is a translation with rotation. This result is known as :

(1) Euler's theorem
(2) Law of inertia
(3) Chasle's theorem
(4) Law of force

66. Two lines of regressions are $X = -\frac{1}{18}Y + \lambda$ and $Y = -2X + \mu$; (λ, μ) being unknown and the mean of the distribution is $(-1, 2)$. Estimated value of X when $Y = 10$ is :

(1) -2 (2) 2

(3) $-\frac{13}{9}$ (4) $\frac{13}{9}$

67. If partial correlation coefficient $r_{12.3} = 0$, then :

(1) $r_{12} = r_{13}r_{23}$ (2) $r_{23} = r_{21}r_{13}$

(3) $r_{31} = r_{12}r_{23}$ (4) $r_{12} = 1$

68. If $(X, Y) \sim B \vee N\,(0, 0, 1, 1, 0.8)$, then $1 + 2X + 3Y$ is distributed as :

(1) $N(0, 1)$ (2) $N(1, 13)$ (3) $N(1, 19)$ (4) $N(0, 19)$

69. In case of simple random sampling with replacement, the variance of the estimate of population mean is :

(1) $\frac{N-n}{nN}\sigma^2$ (2) $\frac{N-n}{nN} \cdot \frac{N-1}{N}\sigma^2$

(3) $\frac{N-n}{nN} \cdot \frac{N}{N-1}\sigma^2$ (4) $\frac{\sigma^2}{n}$

70. Which of the following statements is ***false*** ?

(1) Mean lies in between median and mode

(2) Mean, median and mode have the same unit

(3) In a moderately asymmetrical distribution, Mode = 3 Median – 2 Mean

(4) The median is not affected by the extreme values

71. The following LPP has the multiple optimal solutions :

Max. $Z = x + 3y$

Subject to

$$2x + y \leq 10$$

$$x + 3y \leq 15$$

$$x, y \geq 0$$

One of the points that gives optimal solution for the LPP is :

(1) (5, 0) (2) (2.7, 4.1) (3) (9, 2) (4) (2, 1)

72. With 0.8 as the traffic intensity, the expected number of customers in M | M | 1 system is :

(1) 4 (2) 3.2 (3) 5 (4) $\frac{20}{9}$

73. Number of observations saved in a 4×4 L. S. D. over a complete 3-way layout is :

(1) 4 (2) 12 (3) 24 (4) 48

74. A system is composed of three identical independent elements in series, each having the reliability 0.3, then reliability of the system is :

(1) 0.9 (2) 0.973 (3) 0.027 (4) 0.73

75. While solving an LPP by simplex method to find which variable to leave, ratio column is calculated. If all the ratios turn to be negative or undefined, it indicates the problem has :

(1) Degenerate solution (2) Unbounded solution

(3) No feasible solution (4) Multiple optimal solution

76. Row heading of a statistical table is known as :

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77. Mean deviation is minimum when deviations are taken from :

(1) Mean (2) Median (3) Mode (4) Zero

78. A can hit target 2 times in 5 shots, B 3 times in 5 shots and C 4 times in 5 shots. They fire a volley (each try once to hit the target). The probability that two shots hit the target is :

(1) $\frac{24}{125}$ (2) $\frac{67}{125}$ (3) $\frac{121}{125}$ (4) $\frac{58}{125}$

79. The joint probability density function of a two-dimensional random variable (X, Y) is given by $f(x,y)=\begin{cases}2 & ; \quad 0<x<1, 0<y<x \\ 0 & ; \quad \text{elsewhere}\end{cases}$. The marginal density function of Y is :

(1) 2 (2) $2y$ (3) $2(1-y)$ (4) $2(y-1)$

80. For two random variable X and Y, the relation $E(XY) = E(X)\,E(Y)$ holds good :

(1) if X and Y are statistically independent

(2) for all X and Y

(3) if X and Y are identical

(4) if $X = Y$

81. A function $f(x)$ is a monotonic function if $f(x)$ is :

(1) either increasing or decreasing function

(2) only increasing function

(3) only decreasing function

(4) a constant function

82. The integral $\int_0^{\infty} \frac{\sin x}{x}\,dx$:

(1) converges absolutely (2) does not converge

(3) converges but not absolutely (4) does not exist

83. Let $f_n(x) = nxe^{-nx^2}, x \in [0,1]$, then which of the following is ***not*** a point sequence ?

(1) $a_n = 2ne^{-2n}$ (2) $a_n = ne^{-n}$ (3) $a_n = 2ne^{-4n}$ (4) $a_n = 0$

84. If $f(x,y)=\begin{cases}\dfrac{xy^2}{x^2+y^4} & , (x,y)\neq(0,0)\\ 0 & , (x,y)=(0,0)\end{cases}$, then the directional derivative along $u=\left(\sqrt{2},\sqrt{2}\right)$ at (0, 0) is :

(1) $\sqrt{2}$ (2) $\dfrac{1}{\sqrt{2}}$ (3) $2\sqrt{2}$ (4) $\dfrac{1}{2\sqrt{2}}$

85. If $f : X \to R$, $X \subseteq R^2$ and $(a, b) \in X$ is such that f_x, f_y are differentiable at (a, b), then $f_{xy}(a,b)=f_{yx}(a,b)$. This result is known as :

(1) Schwarz's theorem (2) Young's theorem

(3) Taylor's theorem (4) Implicit function theorem

86. The function $f(x,y)=x^4+x^2y+y^2$ is :

(1) minimum at (0, 0)

(2) neither minimum nor maximum at (0, 0)

(3) maximum at (0, 0)

(4) discontinuous at (0, 0)

87. If $<E_i>$ is a sequence of Lebesgue measurable sets and m is the Lebesgue measure, then :

(1) $m(\cup E_i)=\sum mE_i$ (2) $m(\cup E_i)\geq\sum mE_i$

(3) $m(\cup E_i)\leq\sum mE_i$ (4) $m(\cup E_i)=0$

88. If $\alpha > 0$ and $\beta > 0$ and $f(x)=\begin{cases}x^\alpha \sin\dfrac{1}{x^\beta} & , x\neq 0\\ 0 & , x=0\end{cases}$, then $f(x)$ is of bounded variation in [0, 1] if :

(1) $\alpha < \beta$ (2) $1+\alpha < \beta$ (3) $\alpha + \beta = 1$ (4) $\alpha > \beta$

89. Let $X = \{x : 0 < d(0, x) \leq 1$, and $x \in R^2\}$ where $0 = (0, 0)$ and d is the usual metric on X, then which of the following is ***not*** true ?

(1) X is closed (2) X is bounded

(3) X is not compact (4) X is compact

90. If $X = [X_1\ X_2 \ldots\ldots X_n]'$ is an n-tuple non-zero vector, then the $n \times n$ matrix $V = XX'$:

(1) has rank zero (2) has rank 1 (3) has rank n (4) is orthogonal

91. Solution of $\frac{dy}{dx} = \frac{y}{x} + x \tan \frac{y}{x}$ is :

(1) $\log \cot \frac{y}{x} = x + c$

(2) $\log \tan \frac{y}{x} = x + c$

(3) $\log \cot \frac{y}{2x} = x + c$

(4) $\log \tan \frac{y}{2x} = x + c$

92. Solution of $y = xp - p^2$ is :

(1) $y = \log x + c$ (2) $y = cx - c^2$ (3) $y = 3x - c$ (4) $y = cx - cx^2$

93. P. I. of $(D^2 - 6D + 9)y = 8e^{3x}$ is :

(1) $4xe^{3x}$ (2) $\frac{x}{2}e^{3x}$ (3) $2x^2e^{3x}$ (4) $4x^2e^{3x}$

94. Solving $y'' - 2y' + y = e^x \log x$ by variation for parameters, the value of Wronskion is :

(1) e^{-2x} (2) e^{2x} (3) xe^{2x} (4) $x\bar{e}^{2x}$

95. Green's function of the boundary value problem $y'' = 0, y(0) = y(1) = 0$ is given by :

(1) $G(x, t) = x(1 - t); 0 \leq x < t$

(2) $G(x, t) = x\,t; t < x \leq 1$

(3) $G(x, t) = x^2 (1 - t); 0 \leq x < t$

(4) $G(x, t) = x(1 - t^2); t < x \leq 1$

96. The general solution of $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ is of the form :

(1) $u = cf(x - iy)$

(2) $u = cf(x + y)$

(3) $u = f(x + y) + g(x - y)$

(4) $u = f(x + iy) + g(x - iy)$

97. The initial value problem $u_x + u_y = 1, u(s, s) = \sin s, 0 \leq s \leq 1$ has :

(1) no solution

(2) a unique solution

(3) two solutions

(4) infinitely many solutions

98. The region in which the differential equation $yu_{xx} + 2xyu_{xy} + xu_{yy} = u_x + u_y$, is hyperbolic, is :

(1) $xy > 0$ (2) $xy \neq 0$ (3) $xy > 1$ (4) $xy \neq 1$

99. The partial differential equation formed by eliminating arbitrary function from the equation $z = f(x^2 - y^2)$ is :

(1) $xp + yq = 0$ (2) $xq + yp = 0$ (3) $\frac{x}{y} = q$ (4) $\frac{x}{y} = p$

100. P. I. of $(2D^2 - 3DD' + D'^2)z = e^{x+2y}$ is :

(1) x^2e^{x+2y} (2) xe^{x+2y} (3) $-\frac{x}{3}e^{x+2y}$ (4) $-\frac{x}{2}e^{x+2y}$

Professor & Head
Department of Mathematics
M.D. University, ROHTAK

Total No. of Printed Pages : 17

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M.Phil./Ph.D./URS-EE-Oct.-2017

SUBJECT : Mathematics

D

10216

Sr. No. ....

Time : **1¼ Hours** Max. Marks : **100** Total Questions : **100**

Roll No. (in figures)_______________ (in words) _______________

Name _______________ Father's Name_______________

Mother's Name_______________ Date of Examination_______________

_______________ (Signature of the Candidate)

_______________ (Signature of the Invigilator)

SEAL

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1. ***Candidates are required to attempt any 75 questions out of the given 100 multiple choice questions of 4/3 marks each. No credit will be given for more than 75 correct responses.***
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3. In case there is any discrepancy in any question(s) in the Question Booklet, the same may be brought to the notice of the Controller of Examinations in writing **within two hours** after the test is over. No such complaint(s) will be entertained thereafter.
4. The candidate ***must not*** do any rough work or writing in the OMR Answer-Sheet. Rough work, if any, may be done in the question booklet itself. Answers ***must not*** be ticked in the question booklet.
5. ***There will be no negative marking. Each correct answer will be awarded 4/3 marks. Cutting, erasing, overwriting and more than one answer in OMR Answer-Sheet will be treated as incorrect answer.***
6. Use only **Black** or **Blue Ball Point Pen** of good quality in the OMR Answer-Sheet.
7. *Before answering the questions, the candidates should ensure that they have been supplied correct and complete booklet. Complaints, if any, regarding misprinting etc. will not be entertained* **30** *minutes after starting of the examination.*

1. Solution of Volterra integral equation $y(x) = 1 + x + \int_0^x (x-t)\, y(t)\, dt$, is :

(1) $y = e^x$ (2) $y = e^{-x}$ (3) $y = xe^x$ (4) $y = 1 + e^x$

2. Any non-trivial solution of the homogeneous integral equation for a certain value of λ is called :

(1) eigen value (2) eigen function (3) kernel (4) resolvent kernel

3. The Lagrangian of a particle moving in a plane under the influence of a central potential is given by $L = \frac{1}{2} m\left(\dot{r}^2 + r^2 \dot{\theta}^2\right) - V(r)$. The generalized momenta corresponding to r and θ are given by :

(1) $m\dot{r}$ and $mr\,\dot{\theta}$
(2) $m\dot{r}^2$ and $mr\,\dot{\theta}$
(3) $m\dot{r}^2$ and $mr^2\dot{\theta}$
(4) $m\dot{r}$ and $mr^2\dot{\theta}$

4. If the generalized coordinate is angle θ, the corresponding generalized force has the dimensions of :

(1) velocity
(2) acceleration
(3) force
(4) displacement

5. The generalized displacement of a rigid body is a translation with rotation. This result is known as :

(1) Euler's theorem
(2) Law of inertia
(3) Chasle's theorem
(4) Law of force

6. Two lines of regressions are $X = -\frac{1}{18} Y + \lambda$ and $Y = -2X + \mu$; (λ, μ) being unknown and the mean of the distribution is $(-1, 2)$. Estimated value of X when $Y = 10$ is :

(1) -2
(2) 2
(3) $-\frac{13}{9}$
(4) $\frac{13}{9}$

7. If partial correlation coefficient $r_{12.3} = 0$, then :

(1) $r_{12} = r_{13}r_{23}$ (2) $r_{23} = r_{21}r_{13}$

(3) $r_{31} = r_{12}r_{23}$ (4) $r_{12} = 1$

8. If (X, Y) ~ $B \vee N$ (0, 0, 1, 1, 0.8), then 1 + 2X + 3Y is distributed as :

(1) $N(0, 1)$ (2) $N(1, 13)$ (3) $N(1, 19)$ (4) $N(0, 19)$

9. In case of simple random sampling with replacement, the variance of the estimate of population mean is :

(1) $\frac{N-n}{nN}\sigma^2$ (2) $\frac{N-n}{nN} \cdot \frac{N-1}{N}\sigma^2$

(3) $\frac{N-n}{nN} \cdot \frac{N}{N-1}\sigma^2$ (4) $\frac{\sigma^2}{n}$

10. Which of the following statements is *false* ?

(1) Mean lies in between median and mode

(2) Mean, median and mode have the same unit

(3) In a moderately asymmetrical distribution, Mode = 3 Median – 2 Mean

(4) The median is not affected by the extreme values

11. Solution of $\frac{dy}{dx} = \frac{y}{x} + x\tan\frac{y}{x}$ is :

(1) $\log\cot\frac{y}{x} = x + c$ (2) $\log\tan\frac{y}{x} = x + c$

(3) $\log\cot\frac{y}{2x} = x + c$ (4) $\log\tan\frac{y}{2x} = x + c$

12. Solution of $y = xp - p^2$ is :

(1) $y = \log x + c$ (2) $y = cx - c^2$ (3) $y = 3x - c$ (4) $y = cx - cx^2$

13. P. I. of $(D^2 - 6D + 9)y = 8e^{3x}$ is :

(1) $4xe^{3x}$ (2) $\frac{x}{2}e^{3x}$ (3) $2x^2e^{3x}$ (4) $4x^2e^{3x}$

14. Solving $y'' - 2y' + y = e^x \log x$ by variation for parameters, the value of Wronskion is :

(1) e^{-2x} (2) e^{2x} (3) xe^{2x} (4) $x\bar{e}^{2x}$

15. Green's function of the boundary value problem $y'' = 0, y(0) = y(1) = 0$ is given by :

(1) $G(x, t) = x(1 - t); 0 \le x < t$ (2) $G(x, t) = x\,t; t < x \le 1$

(3) $G(x, t) = x^2(1 - t); 0 \le x < t$ (4) $G(x, t) = x(1 - t^2); t < x \le 1$

16. The general solution of $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ is of the form :

(1) $u = cf(x - iy)$ (2) $u = cf(x + y)$

(3) $u = f(x + y) + g(x - y)$ (4) $u = f(x + iy) + g(x - iy)$

17. The initial value problem $u_x + u_y = 1, u(s, s) = \sin s, 0 \le s \le 1$ has :

(1) no solution (2) a unique solution

(3) two solutions (4) infinitely many solutions

18. The region in which the differential equation $yu_{xx} + 2xyu_{xy} + xu_{yy} = u_x + u_y$, is hyperbolic, is :

(1) $xy > 0$ (2) $xy \ne 0$ (3) $xy > 1$ (4) $xy \ne 1$

19. The partial differential equation formed by eliminating arbitrary function from the equation $z = f(x^2 - y^2)$ is :

(1) $xp + yq = 0$ (2) $xq + yp = 0$ (3) $\frac{x}{y} = q$ (4) $\frac{x}{y} = p$

20. P. I. of $(2D^2 - 3DD' + D'^2)z = e^{x+2y}$ is :

(1) x^2e^{x+2y} (2) xe^{x+2y} (3) $-\frac{x}{3}e^{x+2y}$ (4) $-\frac{x}{2}e^{x+2y}$

21. The function $f(z)=\overline{z}$ is :
(1) Everywhere differentiable
(2) Nowhere differentiable
(3) Differentiable only at $z=0$
(4) Differentiable everywhere except at $z=0$

22. $\int_r \frac{1}{z}\,dz$, where $r(t)=\sin t + i\cos t,\ 0 \le t \le 2\pi$, is :
(1) -2π (2) $4\pi i$ (3) $2\pi i$ (4) $-2\pi i$

23. Using Cauchy's integral formula for derivatives $\int_C \frac{\sin z}{z^4}\,dz$, where C is the circle $|z|=2$, is :
(1) $\pi i/2$ (2) $\pi i/3$ (3) $-\pi i/3$ (4) $-\pi i/2$

24. For the function $f(z)=z^3\sin\left(\frac{1}{z}\right)$, the point $z=0$ is :
(1) zero of order one
(2) pole of order one
(3) essential singularity
(4) zero of order two

25. Residue of $f(z)=e^{3/z}$ at $z=0$, is :
(1) $\frac{1}{3}$ (2) 3 (3) e^3 (4) ∞

26. Using Cauchy Residue theorem, $\oint_C \frac{2z+6}{z^2+4}\,dz$, where C is $|z-i|=2$, is :
(1) $\frac{\pi}{2}(2+3i)$ (2) $\frac{\pi}{2}(3+2i)$ (3) $\pi(2+3i)$ (4) $\pi(3+2i)$

27. The mapping $f(z)=ze^{z^2-2}$ is not conformal at $z=$
(1) $\pm\frac{i}{\sqrt{2}}$ (2) $\pm\frac{1}{\sqrt{2}}$ (3) $\pm\sqrt{2}i$ (4) 0

28. If 20 dictionaries in a library contain 41727 pages, then one of the dictionaries must have at least :
(1) 2084 pages (2) 2085 pages (3) 2086 pages (4) 2087 pages

29. In how many ways can a person can invite his 5 friends for dinner ?

(1) 31 (2) 32 (3) 33 (4) 120

30. For any integer $n > 2$, which of the following is true for the Euler's function $\phi(n)$?

(1) $\phi(n)$ is zero
(2) $\phi(n)$ is even
(3) $\phi(n)$ is odd
(4) $\phi(n)$ is rational number

31. A function $f(x)$ is a monotonic function if $f(x)$ is :

(1) either increasing or decreasing function

(2) only increasing function

(3) only decreasing function

(4) a constant function

32. The integral $\int_0^\infty \frac{\sin x}{x} dx$:

(1) converges absolutely
(2) does not converge
(3) converges but not absolutely
(4) does not exist

33. Let $f_n(x) = nxe^{-nx^2}, x \in [0,1]$, then which of the following is ***not*** a point sequence ?

(1) $a_n = 2ne^{-2n}$ (2) $a_n = ne^{-n}$ (3) $a_n = 2ne^{-4n}$ (4) $a_n = 0$

34. If $f(x,y) = \begin{cases} \frac{xy^2}{x^2+y^4} & , (x,y) \neq (0,0) \\ 0 & , (x,y) = (0,0) \end{cases}$, then the directional derivative along $u = \left(\sqrt{2}, \sqrt{2}\right)$ at (0, 0) is :

(1) $\sqrt{2}$ (2) $\frac{1}{\sqrt{2}}$ (3) $2\sqrt{2}$ (4) $\frac{1}{2\sqrt{2}}$

35. If $f : X \to R$, $X \subseteq R^2$ and $(a, b) \in X$ is such that f_x, f_y are differentiable at (a, b), then $f_{xy}(a,b) = f_{yx}(a,b)$. This result is known as :

(1) Schwarz's theorem
(2) Young's theorem
(3) Taylor's theorem
(4) Implicit function theorem

36. The function $f(x,y)=x^4+x^2y+y^2$ is :

(1) minimum at (0, 0)

(2) neither minimum nor maximum at (0, 0)

(3) maximum at (0, 0)

(4) discontinuous at (0, 0)

37. If $<E_i>$ is a sequence of Lebesgue measurable sets and m is the Lebesgue measure, then :

(1) $m(\cup E_i)=\sum mE_i$ (2) $m(\cup E_i)\geq\sum mE_i$

(3) $m(\cup E_i)\leq\sum mE_i$ (4) $m(\cup E_i)=0$

38. If $\alpha>0$ and $\beta>0$ and $f(x)=\begin{cases} x^{\alpha}\sin\frac{1}{x^{\beta}} & , x\neq 0 \\ 0 & , x=0 \end{cases}$, then $f(x)$ is of bounded variation in [0, 1] if :

(1) $\alpha<\beta$ (2) $1+\alpha<\beta$ (3) $\alpha+\beta=1$ (4) $\alpha>\beta$

39. Let $X=\{x:0<d(0,x)\leq 1, \text{ and } x\in R^2\}$ where $0=(0,0)$ and d is the usual metric on X, then which of the following is ***not*** true ?

(1) X is closed (2) X is bounded

(3) X is not compact (4) X is compact

40. If $X=[X_1\ X_2 ......X_n]'$ is an n-tuple non-zero vector, then the $n\times n$ matrix $V=XX'$:

(1) has rank zero (2) has rank 1 (3) has rank n (4) is orthogonal

41. A symmetric die is thrown 600 times. The lower bound for the probability of getting 80 to 120 sixes is :

(1) $\frac{19}{24}$ (2) $\frac{3}{4}$ (3) $\frac{1}{2}$ (4) $\frac{5}{24}$

42. The interval between two successive occurrence of a Poisson process $\{N(t),\ t \geq 0\}$ having parameter μ has :

(1) Poisson distribution with mean $\frac{1}{\mu}$

(2) Negative exponential distribution with mean $\frac{1}{\mu}$

(3) Negative exponential distribution with mean μ

(4) Poisson distribution with mean μ

43. For a binomial variate X, the mean is 6 and the standard deviation $\sqrt{2}$, then $P(X = 0)$ is :

(1) $\left(\frac{1}{3}\right)^6$

(2) $\left(\frac{2}{3}\right)^6$

(3) $\left(\frac{1}{3}\right)^9$

(4) $\left(\frac{2}{3}\right)^{18}$

44. Let $X_1, X_2, \ldots\ldots, X_n$ be n i.i.d. variates each with p.d.f. $f(x)$ and c. d. f. $F(x)$. The p.d.f. of the smallest order statistic is :

(1) $n[F(x)]^{n-1} \cdot f(x)$

(2) $1-[1-F(x)]^n$

(3) $[F(x)]^n$

(4) $n[1-F(x)]^{n-1} \cdot f(x)$

45. If for a Poisson variate X, $E(X^2) = 6$, then $E(x)$ is :

(1) 6

(2) 3

(3) 2

(4) 1

46. In case of large sample single tailed test, the magnitude of the critical value of z at 5 percent level of significance is :

(1) 2.58

(2) 2.33

(3) 1.96

(4) 1.645

47. The hypothesis that the population variance has a specified value can be tested by :

(1) F-test (2) Z-test

(3) χ^2-test (4) t-test

48. Which of the following is ***not*** true ?

(1) Probability of Type I error is also referred to as consumer's risk

(2) Every most powerful critical region is necessarily unbiased

(3) The standard deviation of the sampling distribution of a statistic is known as its standard error

(4) The value of test statistic which separates the critical region and acceptance region is called critical value

49. Which of the following assumptions is ***not*** associated with non-parametric tests ?

(1) Sample observations are independent

(2) Parent population from which the sample(s) have been drawn is normal

(3) The variate under study is continuous

(4) p. d. f. is continuous

50. Non-parametric test to be used for testing if two independent ordered samples differ in their central tendencies is :

(1) Sign test (2) Run test

(3) Median test (4) Mann-Whitney-Wilcoxon U-test

51. $\left(\frac{\Delta^2}{E}\right)e^x \cdot \frac{Ee^x}{\Delta^2 e^x} =$

(1) e^x (2) xe^x (3) $(x+1)e^x$ (4) $\frac{e^x}{x}$

52. Consider the series $x_{n+1} = \frac{x_n}{2} + \frac{9}{8x_n}, x_0 = \frac{1}{2}$ obtained from Newton-Raphson method. This series converges to :

(1) $\sqrt{2}$ (2) $\frac{2}{3}$ (3) $\frac{3}{2}$ (4) $\frac{8}{5}$

53. In solving the differential equation $y' = 2x, y(0) = 0$ using Euler's method, the interacts $y_n, n \in N$ satisfy :

(1) $y_n = 2x_n$

(2) $y_n = 2x_n - x_{n-1}$

(3) $y_n = x_n + x_{n-1}$

(4) $y_n = x_n x_{n-1}$

54. Hermite's interpolation formula is also called :

(1) osculating interpolation formula

(2) constant interpolation formula

(3) increasing interpolation formula

(4) decreasing interpolation formula

55. In Runge-Kutta fourth order method, for the initial value problem $\frac{dy}{dx} = f(x, y)$, $y(x_0) = y_0$ approximate value of y is given by $y_1 = y_o + k$, where with usual notations :

(1) $k = \frac{1}{2}(k_1 + 2k_2 + 2k_3 + k_4)$

(2) $k = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

(3) $k = \frac{1}{4}(k_1 + k_2 + k_3 + k_4)$

(4) $k = \frac{1}{6}(k_1 + 3k_2 + 3k_3 + k_4)$

56. The values of a, b, c respectively for which the formula $\int_a^b f(x)dx = h\left[af(0) + bf\left(\frac{h}{3}\right) + cf(h)\right]$ is exact for polynomials of as higher order as possible, are :

(1) $0, \frac{3}{4}, \frac{1}{4}$ (2) $0, \frac{1}{4}, \frac{3}{4}$ (3) $\frac{3}{4}, 0, \frac{1}{4}$ (4) $\frac{1}{4}, 0, \frac{3}{4}$

57. On what curve the function $I = \int_0^1 \left[\left(\frac{dy}{dx}\right)^2 + 12xy\right] dx$ with $y(0) = 0$, $y(1) = 1$ can be extremized ?

(1) $y = x$ (2) $y = x^2$ (3) $y = x^3$ (4) $y = x^4$

58. Rayleigh-Ritz method is used to :

(1) find maxima (2) find geodesics

(3) find minima (4) solve boundary value problem

59. The simplification of the Euler-Lagrange equation is known as :

(1) Beltrami identity (2) Liouveille's identity

(3) Hamilton identity (4) Cauchy identity

60. The solution of the integral equation $\int_0^x \frac{y(t)}{x-t} dt = \sqrt{x}$ is :

(1) $y = 1$ (2) $y = \frac{3}{2}$ (3) $y = \frac{1}{2}$ (4) $y = \frac{3}{4}$

61. The following LPP has the multiple optimal solutions :

Max. $Z = x + 3y$

Subject to

$2x + y \leq 10$

$x + 3y \leq 15$

$x, y \geq 0$

One of the points that gives optimal solution for the LPP is :

(1) (5, 0) (2) (2.7, 4.1) (3) (9, 2) (4) (2, 1)

62. With 0.8 as the traffic intensity, the expected number of customers in M | M | 1 system is :

(1) 4 (2) 3.2 (3) 5 (4) $\frac{20}{9}$

63. Number of observations saved in a 4×4 L. S. D. over a complete 3-way layout is :

(1) 4 (2) 12 (3) 24 (4) 48

64. A system is composed of three identical independent elements in series, each having the reliability 0.3, then reliability of the system is :

(1) 0.9 (2) 0.973 (3) 0.027 (4) 0.73

65. While solving an LPP by simplex method to find which variable to leave, ratio column is calculated. If all the ratios turn to be negative or undefined, it indicates the problem has :

(1) Degenerate solution
(2) Unbounded solution
(3) No feasible solution
(4) Multiple optimal solution

66. Row heading of a statistical table is known as :

(1) sub-title
(2) stub
(3) caption
(4) reference note

67. Mean deviation is minimum when deviations are taken from :

(1) Mean
(2) Median
(3) Mode
(4) Zero

68. A can hit target 2 times in 5 shots, B 3 times in 5 shots and C 4 times in 5 shots. They fire a volley (each try once to hit the target). The probability that two shots hit the target is :

(1) $\frac{24}{125}$ (2) $\frac{67}{125}$ (3) $\frac{121}{125}$ (4) $\frac{58}{125}$

69. The joint probability density function of a two-dimensional random variable (X, Y) is given by $f(x,y)=\begin{cases}2 \ ; & 0<x<1, 0<y<x \\ 0 \ ; & \text{elsewhere}\end{cases}$. The marginal density function of Y is :

(1) 2 (2) $2y$ (3) $2(1-y)$ (4) $2(y-1)$

70. For two random variable X and Y, the relation $E(XY) = E(X)\,E(Y)$ holds good :

(1) if X and Y are statistically independent

(2) for all X and Y

(3) if X and Y are identical

(4) if $X = Y$

71. The generators of the group $G = \{a, a^2, a^3, a^4, a^5, a^6 = e\}$ are :

(1) a and a^5 (2) a^3 and a^5 (3) a^2 and a^3 (4) a and a^3

72. If N is the set of natural numbers, then under the binary operation $a \,.\, b = a + b, (N, .)$ is :

(1) group (2) semi-group (3) quasi-group (4) monoid

73. If $C = (1, 2, 3, 4)$, then C^2 is :

(1) (1, 4) (2, 3) (2) (1, 3) (3) (1, 3) (2, 4) (4) (2, 4)

74. Which of the following is ***not*** true ?

The relation of isomorphism in the set of all groups :

(1) satisfies reflexivity (2) satisfies anti-symmetry

(3) satisfies transitivity (4) is an equivalence relation

75. $\left[Q(\sqrt{2}, \sqrt{3}) : Q\right] =$

(1) 4 (2) 3 (3) 2 (4) 1

76. Which of the following is ***not*** a field ?

(1) $\frac{z}{2z}$ (2) $\frac{z}{3z}$ (3) $\frac{z}{4z}$ (4) $\frac{z}{5z}$

77. Given a field F and the set M of all 2×2 matrices of the form $\begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix}$ for $a, b \in F$, then which of the following is *not* true ?

(1) M is left ideal in F

(2) M is right ideal in F

(3) M is subring of F

(4) M is right ideal but not left ideal in F

78. Which of the following is *not* true ?

(1) Every metric space is first countable

(2) A metric space is second countable iff it is separable

(3) Every metric space is second countable

(4) Any open subspace of a separable space is separable

79. Which of the following is *not* correct ?

(1) Every compact metric space is complete

(2) A compact Hausdorff space is normal

(3) A compact subspace of a Hausdorff space is closed

(4) Every metric space is compact Hausdorff space

80. If $X = \{a, b, c\}$, $T = \{\phi, X, \{a, c\}, \{b\}\}$, then the topological space (X, T) is :

(1) not a connected space

(2) A connected space

(3) not a Hausdorff space

(4) not a compact space

81. For the matrix $A = \begin{bmatrix} 2 & -2 & 3 \\ -2 & -1 & 6 \\ 1 & 2 & 0 \end{bmatrix}$, one of the eigen values is 3, the other two eigen values are :

(1) 2, –5

(2) 3, 5

(3) 3, –5

(4) 2, 5

82. Let V be the vector space of ordered pairs of complex numbers over the real field R, then the dimension of V is :

(1) 6

(2) 4

(3) 2

(4) 1

83. Let T be a linear transformation on a vector space V such that $T^2 - T + 1 = 0$, then T is :

(1) singular (2) invertible (3) not invertible (4) idempotent

84. A real quadratic form $X^T AX$ in three variables is equivalent to the diagonal form $3y_1^2 - 4y_2^2 + 5y_3^2$. Then, the quadratic form $X^T AX$ is :

(1) positive definite (2) negative definite

(3) positive semi-definite (4) indefinite

85. The orthogonal complement of inner product space V is :

(1) zero subspace (2) any subspace

(3) V itself (4) None of these

86. If $z = -\sqrt{3} - i$, then a value of z^4 is :

(1) $8(1+i\sqrt{3})$ (2) $8(1-i\sqrt{3})$ (3) $8(-1+i\sqrt{3})$ (4) $8(\sqrt{3}-i)$

87. If $w = u + iv$, $z = x + iy$, then the image of the line $x = -3$ under the complex mapping $w = z^2$ is :

(1) $u = -3v$ (2) $u = -3 + \frac{v^2}{4}$ (3) $v = 9 - \frac{u^2}{36}$ (4) $u = 9 - \frac{v^2}{36}$

88. Value of $i^{1/3}$ is :

(1) $\frac{\sqrt{3}}{2} - \frac{1}{2}i$ (2) $\frac{\sqrt{3}}{2} + \frac{1}{2}i$ (3) $\frac{1}{2} + \frac{\sqrt{3}}{2}i$ (4) $\frac{1}{2} + \frac{1}{2}i$

89. Solution of $e^{z-1} = -ie^3$ is :

(1) $4 + (2n - 1)\pi i$ (2) $4 + \frac{1}{2}(2n - 1)\pi i$

(3) $4 + \frac{1}{2}(4n - 1)\pi i$ (4) $\frac{1}{4}(2n - 1)\pi i$

90. Solution of the equation $\cos z = i \sin z$ is :

(1) $z = \frac{2(n+1)}{3}\pi i$ (2) $z = (4n+1)\frac{\pi i}{2}$

(3) $\left(\frac{2n+1}{2}\right)\pi i$ (4) No solution

91. If $n(A) = 115$, $n(B) = 225$, $n(A - B) = 73$, then $n(A \cup B) =$

(1) 265 (2) 278 (3) 295 (4) 298

92. The set of interior points of which of the following sets is ***not*** empty ?

(1) R (2) N (3) Z (4) I

93. If X is the set of even natural numbers less than 8 and Y is the set of odd prime numbers less than equal to 7, then the number of relations from X to Y is :

(1) 9 (2) $2^9 - 1$ (3) 2^{9-1} (4) 2^9

94. The sequence $S_n = \begin{cases} 2 & , \text{ when } n \text{ is even} \\ \text{lowest prime factor } (\neq 1) \text{ of } n & , \text{ when } n \text{ is odd} \end{cases}$ has limit point :

(1) 2

(2) countable in number

(3) 1, 2, 3, 4, ...

(4) uncountable in number

95. The series $\sum_{n=1}^{\infty} \frac{1}{\left(1+\frac{1}{n}\right)^{n^2}}$ is :

(1) convergent

(2) conditionally convergent

(3) divergent

(4) oscillatory

96. $\lim_{n\to\infty}\left[\frac{n}{n^2}+\frac{n}{(n^2+1^2)}+\frac{n}{(n^2+2^2)}+......\right]=$

(1) 0 (2) $\frac{\pi}{3}$ (3) $\frac{\pi}{4}$ (4) $\frac{-\pi}{4}$

97. $\lim_{x\to 0}\frac{\sinh x - \sin x}{x \sin^2 x} =$

(1) $\frac{1}{6}$ (2) $\frac{1}{4}$ (3) $\frac{1}{3}$ (4) $\frac{1}{2}$

98. Let $f(x)=\begin{cases}\dfrac{e^{1/x}}{1+e^{1/x}} & , \quad x\neq 0\\ 0 & , \quad x=0\end{cases}$, then which of the following is true ?

(1) $f(x)$ is continuous at $x = 0$

(2) $f(x)$ is discontinuous at $x = 0$

(3) $f(x)$ is differentiable at $x = 0$

(4) $f(x)$ has discontinuity of first kind from left at $x = 0$

99. If $f(x + y) = f(x) f(y) \ \forall \ x, y$ and $f(5) = -2$ and $f'(0) = 3$, then the value of $f'(5) =$

(1) –3 (2) –5 (3) –6 (4) 6

100. What is the abscissa of the point at which the tangent to the curve $y = x\,(x - 1)$ is parallel to the chord joining the extrimities of the curve in the interval [1, 2] ?

(1) 5/4 (2) 5/3 (3) 4/3 (4) 3/2

Professor & Head
Department of Mathematics
M.D. University, ROHTAK

M.PHIL/PH.D/URS (MATHMEATICS)

PAGE: 1

SET : A

1. 4	16. 1	31. 2	46. 3	61. 1	76. 3	91. 1
2. 1	17. 3	32. 4	47. 1	62. 3	77. 1	92. 2
3. 4	18. 4	33. 3	48. 3	63. 4	78. 3	93. 3
4. 2	19. 4	34. 3	49. 4	64. 1	79. 4	94. 4
5. 1	20. 2	35. 2	50. 1	65. 2	80. 1	95. 3
6. 3	21. 3	36. 4	51. 4	66. 1	81. 2	96. 4
7. 3	22. 2	37. 1	52. 2	67. 3	82. 1	97. 3
8. 2	23. 2	38. 4	53. 4	68. 4	83. 4	98. 1
9. 3	24. 4	39. 1	54. 2	69. 1	84. 3	99. 2
10. 4	25. 1	40. 2	55. 1	70. 3	85. 2	100. 3
11. 1	26. 3	41. 1	56. 4	71. 1	86. 2	
12. 3	27. 4	42. 2	57. 4	72. 2	87. 2	
13. 1	28. 2	43. 3	58. 3	73. 4	88. 4	
14. 1	29. 3	44. 2	59. 2	74. 2	89. 3	
15. 2	30. 4	45. 1	60. 4	75. 3	90. 1	

M.PHIL/PH.D/URS (MATHMEATICS)

PAGE: 2

SET : B

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1. 1	16. 4	31. 4	46. 4	61. 1	76. 1	91. 2
2. 3	17. 3	32. 2	47. 1	62. 2	77. 3	92. 1
3. 1	18. 1	33. 4	48. 4	63. 3	78. 4	93. 4
4. 1	19. 2	34. 2	49. 1	64. 2	79. 1	94. 3
5. 2	20. 3	35. 1	50. 2	65. 1	80. 3	95. 2
6. 1	21. 1	36. 4	51. 3	66. 3	81. 4	96. 2
7. 3	22. 2	37. 4	52. 2	67. 1	82. 1	97. 2
8. 4	23. 4	38. 3	53. 2	68. 3	83. 4	98. 4
9. 4	24. 2	39. 2	54. 4	69. 4	84. 2	99. 3
10. 2	25. 3	40. 4	55. 1	70. 1	85. 1	100. 1
11. 1	26. 3	41. 2	56. 3	71. 1	86. 3	
12. 2	27. 1	42. 4	57. 4	72. 3	87. 3	
13. 3	28. 3	43. 3	58. 2	73. 4	88. 2	
14. 4	29. 4	44. 3	59. 3	74. 1	89. 3	
15. 3	30. 1	45. 2	60. 4	75. 2	90. 4	

M.PHIL/PH.D/URS (MATHMEATICS)

PAGE: 3

SET : C

1. 1	16. 3	31. 1	46. 1	61. 1	76. 2	91. 4
2. 2	17. 4	32. 2	47. 3	62. 2	77. 2	92. 2
3. 3	18. 2	33. 3	48. 4	63. 4	78. 4	93. 4
4. 2	19. 3	34. 4	49. 1	64. 2	79. 3	94. 2
5. 1	20. 4	35. 3	50. 3	65. 3	80. 1	95. 1
6. 3	21. 4	36. 4	51. 2	66. 3	81. 1	96. 4
7. 1	22. 1	37. 3	52. 4	67. 1	82. 3	97. 4
8. 3	23. 4	38. 1	53. 3	68. 3	83. 1	98. 3
9. 4	24. 2	39. 2	54. 3	69. 4	84. 1	99. 2
10. 1	25. 1	40. 3	55. 2	70. 1	85. 2	100. 4
11. 3	26. 3	41. 1	56. 4	71. 2	86. 1	
12. 2	27. 3	42. 3	57. 1	72. 1	87. 3	
13. 2	28. 2	43. 4	58. 4	73. 4	88. 4	
14. 4	29. 3	44. 1	59. 1	74. 3	89. 4	
15. 1	30. 4	45. 2	60. 2	75. 2	90. 2	

M.PHIL/PH.D/URS (MATHMEATICS)

SET : D

1. 1	16. 4	31. 1	46. 4	61. 2	76. 3	91. 4
2. 2	17. 4	32. 3	47. 3	62. 1	77. 1	92. 1
3. 4	18. 3	33. 1	48. 1	63. 4	78. 3	93. 4
4. 2	19. 2	34. 1	49. 2	64. 3	79. 4	94. 2
5. 3	20. 4	35. 2	50. 3	65. 2	80. 1	95. 1
6. 3	21. 2	36. 1	51. 1	66. 2	81. 3	96. 3
7. 1	22. 4	37. 3	52. 3	67. 2	82. 2	97. 3
8. 3	23. 3	38. 4	53. 4	68. 4	83. 2	98. 2
9. 4	24. 3	39. 4	54. 1	69. 3	84. 4	99. 3
10. 1	25. 2	40. 2	55. 2	70. 1	85. 1	100. 4
11. 4	26. 4	41. 1	56. 1	71. 1	86. 3	
12. 2	27. 1	42. 2	57. 3	72. 2	87. 4	
13. 4	28. 4	43. 3	58. 4	73. 3	88. 2	
14. 2	29. 1	44. 4	59. 1	74. 2	89. 3	
15. 1	30. 2	45. 3	60. 3	75. 1	90. 4	