Code–A

Question No.	Questions
1.	The non-empty set of real numbers which is bounded below has (1) supremum (2) infimum (3) upper bound (4) none of these
2.	The sequence $\{f_n\}$ where $f_n(x) = x^n$ is ________ convergent on [0, k], k < 1 (1) uniformly (2) pointwise (3) nowhere (4) none of these
3.	Every bounded sequence has at least one limit point. This represents (1) Archimedean Property (2) Heine-Borel theorem (3) Bolzano-Weierstress theorem (4) Denseness Property
4.	Which of the following is convergent ? (1) $\sum_{n=1}^{\infty} n^2 2^{-n}$ (2) $\sum_{n=1}^{\infty} n^{-2} 2^{n}$ (3) $\sum_{n=2}^{\infty} \frac{1}{n \log n}$ (4) $\sum_{n=1}^{\infty} \frac{1}{n \log(1+1/n)}$
5.	If a function f defined on [0, 1] as $f(x) = \begin{cases} i, \text{if } x \neq 1/2 \\ 0, \text{if } x = 1/2 \end{cases}$, then (1) f is not bounded (2) f is R-integrable (3) f is not R-integrable since f is not bounded (4) f is not R-integrable since lower and upper limits are unequal
6.	Let $f : R \to R$ be a monotone function. Then (1) f has no discontinuities (2) f has only finitely many discontinuities. (3) f can have at most countably many discontinuities (4) f can have uncountably many discontinuities

Question No.	Questions
7.	The length of an interval I is (1) Outer measure of an interval I (2) Less than outer measure of an interval I (3) Greater than outer measure of an interval I (4) Twice the outer measure of an interval I
8.	A set E is said to be Lebesgue measurable if for each set A (1) $m^*(A) = m^*(A \cap E) - m^*(A \cap E^c)$ (2) $m^*(A) = m^*(A \cap E^c) - m^*(A \cap E)$ (3) $m^*(A) = m^*(A \cup E) + m^*(A \cap E^c)$ (4) $m^*(A) = m^*(A \cup E) - m^*(A \cap E^c)$
9.	A non-negative measurable function f is integrable over the measurable set E if (1) $\int_E f = \infty$ (2) $\int_E f > \infty$ (3) $\int_E f < \infty$ (4) None of these
10.	If f is of bounded variation on [a, b] and $c \in (a, b)$. Then (1) f is of bounded variation on [a, c] and on [c, b] (2) f is not of bounded variation on [a, c] and on [c, b] (3) f is constant on [a, c] and on [c, b] (4) None of these
11.	Let X and Y be metric spaces, and $f : X \to Y$ a function then which of the following is true (1) f is continuous ; (2) for every open set U in Y, $f^{-1}(U)$ is open in X (3) for every closed set C in Y, $f^{-1}(C)$ is closed in X (4) All the above

Question No.	Questions
12.	The metric space (R, d), where d is a usual metric, is (1) compact (2) disconnected (3) connected but not compact (4) compact and connected
13.	Let (X, d) be a metric space, then for all x, y, z $\in X$ (1) $d(x, y) \leq d(x, z) + d(z, y)$ (2) $d(x, y) \geq d(x, z) + d(z, y)$ (3) $d(x, y) \leq 0$ (4) None of these
14.	A normed linear space X is complete iff (1) Every convergent series in X is absolutely convergent (2) Every convergent series in X is convergent (3) Every convergent series in X is uniformly convergent (4) Every absolutely convergent series in X is convergent
15.	If W_1, W_2 are two subspaces of a finite dimension vector space V(F), then (1) $\dim(W_1 + W_2) = \dim(W_1 \cup W_2)$ (2) $\dim(W_1 + W_2) = \dim W_1 + \dim W_2$ (3) $\dim(W_1 + W_2) = \dim W_1 + \dim W_2 - \dim(W_1 \cap W_2)$ (4) $\dim(W_1 + W_2) = \dim W_1 + \dim W_2 + \dim(W_1 \cap W_2)$
16.	The co-ordinates of vector (1, 1, 1) relative to basis (1, 1, 2), (2, 2, 1), (1, 2, 2) is (1) (1/3, 0, 1/3) (2) (1/3, 1/3, 0) (3) (0, 1/3, 1/3) (4) None of these
17.	Let $T : R^n \to R^n$ be a linear transformation. Which of the following statements implies that T is bijective ? (1) Nullity (T) = n (2) Rank (T) = Nullity (T) = n (3) Rank (T) + Nullity (T) = n (4) Rank (T) – Nullity (T) = n

Question No.	Questions
18.	Let A, B be n × n real matrices. Which of the following statements is correct ? (1) rank (A + B) = rank(A) + rank(B) (2) rank (A + B) $\leq$ rank(A) + rank(B) (3) rank (A + B) = min {rank(A), rank(B)} (4) rank (A + B) = max {rank (A), rank(B)}
19.	Let A and B be real invertible matrices such that AB = – BA. Then (1) Trace (A) = 1, Trace (B) = 0 (2) Trace (A) = Trace (B) = 1 (3) Trace (A) = 0, Trace (B) = 1 (4) Trace (A) = Trace (B) = 0
20.	Consider the matrix $A(x) = \begin{bmatrix} 1+x^2 & 7 & 11 \\ 3x & 2x & 4 \\ 8x & 17 & 13 \end{bmatrix}$; $x \in R$. Then (1) A (x) has eigenvalue 0 for some $x \in R$ (2) 0 is not an eigenvalue of A(x) for any $x \in R$ (3) A (x) has eigenvalue 0 for all $x \in R$ (4) A (x) is invertible for every $x \in R$
21.	The Linear transformation $T : R^3 \to R^3$ corresponding to the matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ is (1) $T(x_1, x_2, x_3) = (x_1, 2x_2, 3x_3)$ (2) $T(x_1, x_2, x_3) = (x_1 + x_3, 2x_1 + x_2, x_2 + x_3)$ (3) $T(x_1, x_2, x_3) = (x_1, x_2, x_3)$ (4) None of these

Question No.	Questions
22.	The matrix $\begin{bmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix}$ is (1) non-negative definite but not positive definite (2) positive definite (3) negative definite (4) neither negative definite nor positive definite
23.	Let $X = \begin{bmatrix} 2 & 0 & -3 \\ 3 & -1 & -3 \\ 0 & 0 & -1 \end{bmatrix}$. A matrix P such that $P^{-1}XP$ is a diagonal matrix, is (1) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ (2) $\begin{bmatrix} -1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ (3) $\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ (4) $\begin{bmatrix} -1 & -1 & 1 \\ 0 & -1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$
24.	The norm of x with respect to inner product space $\langle x, x \rangle$ is (1) $\lVert x \rVert = \langle x, x \rangle$ (2) $\lVert x \rVert^2 = \langle x, x \rangle$ (3) $\lVert x \rVert = \langle x, x \rangle^2$ (4) None of these
25.	Cayley-Hamilton theorem states that (1) Every square matrix satisfies its own characteristic equation (2) Every square matrix does not satisfy its own characteristic equation (3) Every rectangular matrix satisfies its own characteristic equation (4) None of these

Question No.	Questions
26.	If $\lvert z \rvert = \lvert z-1 \rvert$ then (1) Re (z) = 1 (2) Re (z) = 1/2 (3) Im (z) = 1 (4) Im (z) = 1/2
27.	The power series $\sum_{n=0}^{\infty} 3^{-n} (z-1)^{2n}$ converges if (1) $\lvert z \rvert \le 3$ (2) $\lvert z \rvert < \sqrt{3}$ (3) $\lvert z-1 \rvert < \sqrt{3}$ (4) $\lvert z-1 \rvert \le \sqrt{3}$
28.	An analytic function of a complex variable z = x + iy is expressed as $f(z) = u(x, y) + iv(x, y)$, where $i = \sqrt{-1}$. If u (x, y) = 2 xy, then v (x, y) must be (1) $x^2 + y^2$ + constant (2) $x^2 - y^2$ + constant (3) $-x^2 + y^2$ + constant (4) $-x^2 - y^2$ + constant
29.	$\int_{\lvert z \rvert=2} \frac{2z}{z^2+2} dz =$ (1) 0 (2) $-2\pi i$ (3) $4\pi i$ (4) 1
30.	The value of $\int_C \frac{\sin z}{4z+\pi} dz$ where C : $\lvert z \rvert = 1$ is a positively oriented contour. (1) 0 (2) $\frac{-\sqrt{2}\,\pi i}{4}$ (3) $\frac{-\sqrt{2}\,i}{4}$ (4) $\frac{-\pi i}{4}$

Question No.	Questions
31.	The statement, if f is entire and bounded for all $z \in C$, then f is constant, refers to : (1) Morera's theorem (2) Maximum modulus theorem (3) Liouville's theorem (4) Hurwitz theorem
32.	Let f(z) be analytic in a closed, connected domain, D then which of the following is not true (1) The extreme values of the modulus of the function must occur on the boundary (2) If $\mid f(z) \mid$ has an interior extrema, then the function is a constant (3) The extreme values of the modulus of the function may occur on interior point and f(z) may not be constant (4) If f(z) is non constant then $\mid f(z) \mid$ does not attains an extrema inside the boundary
33.	The function $f : C \to C$ defined by $f(z) = e^z + e^{-z}$ has (1) finitely many zeros (2) no zeros (3) only real zeros (4) has infinitely may zeros
34.	Consider the following complex function $f(z) = \frac{9}{(z-1)(z+2)^2}$. Which of the following is one of the residues of the above function ? (1) –1 (2) 9/16 (3) 2 (4) 9
35.	The bilinear transformation that maps the points $z = \infty$, i, 0 into the points $w = 0$, i, ∞ is (1) $w = -z$ (2) $w = z$ (3) $w = \frac{-1}{z}$ (4) $w = \frac{1}{z}$

Question No.	Questions
36.	The fundamental theorem of arithmetic states that (1) The factoring of any integer $n > 1$ into primes is not unique apart from the order of prime factors (2) The factoring of any integer $n > 1$ into primes is unique apart from the order of prime factors (3) There are infinitely many primes (4) The number of prime numbers is finite
37.	The last two digits of 7^{81} are (1) 07 (2) 17 (3) 37 (4) 47
38.	The congruence $35x \equiv 14 \pmod{21}$ has (1) 7 solutions (2) 6 solutions (3) 9 solutions (4) No solution
39.	If n is a positive integer such that the sum of all positive integers a satisfying $1 \leq a \leq n$ and GCD $(a, n) = 1$ is equal to $240n$, then the number of summands, namely, $\phi(n)$, is (1) 120 (2) 124 (3) 240 (4) 480
40.	If gcd $(m, n) = 1$ where $m > 2$ and $n > 2$, then the integer mn has (1) no primitive roots (2) unique primitive root (3) infinite primitive roots (4) finite primitive roots
41.	If p is a prime, then any group G of order 2p has (1) a normal subgroup of order p (2) a normal subgroup of order 2p (3) a normal subgroup of order p^2 (4) None of these

Question No.	Questions
42.	Let G be simple group of order 60. Then (1) G has six Sylow-5 subgroups (2) G has four Sylow-3 subgroups (3) G has a cyclic subgroup of order 6 (4) G has a unique element of order 2
43.	Let R be a Euclidean domain such that R is not a field. Then the polynomial ring R[X] is always (1) a Euclidean domain (2) a principal ideal domain but not a Euclidean domain (3) a unique factorization domain but not a principal ideal domain (4) not a unique factorization domain
44.	Let $p(x) = 9x^5 + 10x^3 + 5x + 15$ and $q(x) = x^3 - x^2 - x - 2$ be two polynomials in Q[x]. Then over Q, (1) p(x) and q(x) are both irreducible (2) p(x) is reducible but q(x) is irreducible (3) p(x) is irreducible but q(x) is reducible (4) p(x) and q(x) are both reducible
45.	Find the degree of the field extension $Q\left(\sqrt{2}, \sqrt[4]{2}, \sqrt[8]{2}\right)$ over Q. (1) 4 (2) 8 (3) 14 (4) 32
46.	Let F be a finite field and let K/F be a field extension of degree 6. Then the Galois group of K/F is isomorphic to (1) the cyclic group of order 6 (2) the permutation group of {1, 2, 3} (3) the permutation group on {1, 2, 3, 4, 5, 6} (4) the permutation group on {1}

Question No.	Questions
47.	Let X be a topological space and A be a subset of X, then X is separable if (1) A is countable and $\overline{A} = X$ (2) $\overline{A}$ is countable (3) A is uncountable (4) None of these
48.	Which of the following spaces is not separable ? (1) R with the trivial topology (2) The Cantor set as a subspace of R (3) R with the discrete topology (4) None of these
49.	Which of the following is true ? (1) Let X be compact and $f : X \to R$ be locally bounded. Then f is not bounded. (2) Closed subspaces of compact spaces are compact (3) Closed subspaces of compact spaces may not be compact (4) Continuous images of compact spaces may not be compact
50.	Let X and Y be two topological spaces and let $f : X \to Y$ be a continuous function. Then (1) f (K) is connected if $K \subset X$ is connected (2) f^{-1} (K) is connected if $K \subset Y$ is connected (3) f^{-1} (K) is compact if $K \subset Y$ is compact (4) None of these
51.	Consider the initial value problem (IVP) $\frac{dy}{dx} = y^2, \; y(0) = 1, (x, y) \in R \times R.$ Then there exists a unique solution of the IVP on (1) $(-\infty, \infty)$ (2) $(-\infty, 1)$ (3) $(-2, 2)$ (4) $(-1, \infty)$

Question No.	Questions
52.	The solution to the initial value problem $\frac{d^2y}{dt^2}+2\frac{dy}{dt}+5y = 3e^{-t}\sin t$, $y(0) = 0$ and $y'(0) = 3$, is (1) $y(t) = e^{t}(\sin t + \sin 2t)$ (2) $y(t) = e^{-t}(\sin t + \sin 2t)$ (3) $y(t) = 3e^{t}\sin t$ (4) $y(t) = 3e^{-t}\sin t$
53.	Consider the differential equation $(x-1)y'' + xy' + \frac{1}{x}y = 0$. Then (1) $x = 1$ is the only singular point (2) $x = 0$ is the only singular point (3) both $x = 0$ and $x = 1$ are singular points (4) neither $x = 0$ nor $x = 1$ are singular points
54.	Let f and g be real linearly independent solutions of $\frac{d}{dx}\left[\frac{dy}{dx}P(x)\right] + Q(x)y = 0$ on the interval $a \le x \le b$. Then (1) between any two consecutive zeros of f, there is precisely one zero of g. (2) between any two consecutive zeros of f, there is no zero of g. (3) between any two consecutive zeros of f, there is infinite zeros of g. (4) None of these
55.	The eigen values of a Sturm-Liouville BVP are (1) Always positive (2) Always negative (3) Always real (4) Always in the pair of complex conjugate

Question No.	Questions
56.	The Charpit's equations for the PDE $up^2 + q^2 + x + y = 0$, $p = \frac{\partial u}{\partial x}$, $q = \frac{\partial u}{\partial y}$ are given by (1) $\frac{dx}{-1-p^3} = \frac{dy}{-1-qp^2} = \frac{du}{2p^2u+2q^2} = \frac{dp}{2pu} = \frac{dq}{2q}$ (2) $\frac{dx}{2pu} = \frac{dy}{2q} = \frac{du}{2p^2u+2q^2} = \frac{dp}{-1-p^3} = \frac{dq}{-1-qp^2}$ (3) $\frac{dx}{up^2} = \frac{dy}{q^2} = \frac{du}{0} = \frac{dp}{x} = \frac{dq}{y}$ (4) $\frac{dx}{2q} = \frac{dy}{2pu} = \frac{du}{x+y} = \frac{dp}{p^2} = \frac{dq}{qp^2}$
57.	The partial differential equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + u$ can be transformed to $\frac{\partial v}{\partial t} = \frac{\partial^2 v}{\partial x^2}$ for (1) $v = e^{-t} u$. (2) $v = e^{t} u$. (3) $v = tu$. (4) $v = -tu$.
58.	The PDE $x\frac{\partial^2 u}{\partial x^2} + y\frac{\partial^2 u}{\partial y^2} = 0$ is (1) elliptic for $x < 0, y > 0$ (2) hyperbolic for $x > 0, y < 0$ (3) elliptic for $x > 0, y < 0$ (4) hyperbolic for $x > 0, y > 0$

Question No.	Questions
59.	Solution of $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x\, \partial y} - 6\frac{\partial^2 z}{\partial y^2} = 0$ is given by (1) $z = f_1(y + 3x) + f_2(y + 2x)$ (2) $z = f_1(y + 3x) + f_2(y - 2x)$ (3) $z = f_1(y - 2x) + f_2(y + 2x)$ (4) None of these
60.	Given Wave equation $\frac{\partial z}{\partial y} = c^2 \frac{\partial^2 z}{\partial x^2}$. By separation of variable, let $z(x, y) = X(x)\, Y(y)$ be a solution. Substituting it in $\frac{\partial z}{\partial y} = c^2 \frac{\partial^2 z}{\partial x^2}$ one have $\frac{d^2X}{dx^2} + kX = 0$ and $\frac{dY}{dy} - kc^2 Y = 0$. If $k = -p^2$, p is real, then the solution is (c's are constants) (1) $z(x, y) = (c_1 \cos px)\, c_2\, e^{-c^2 p^2 y}$ (2) $z(x, y) = (c_1 \cos px + c_2 \sin px)\, c_1\, e^{-c^2 p^2 y}$ (3) $z(x, y) = e^{-c^2 p^2 y}$ (4) $z(x, y) = (c_2 \cos px)\, c_1\, e^{-c^2 p^2 y}$
61.	The rate of convergence is faster for (1) Regula-Falsi method (2) Bisection method (3) Newton-Raphson method (4) Cannot say
62.	As soon as a new value of a variable is found by iteration, it is used immediately in the following equations, this method is called (1) Gauss-Jordan method (2) Gauss-Seidal method (3) Jacobi's method (4) Relaxation method

Question No.	Questions
63.	The value of function f (x) at 4 discrete points are givne below :

x	0	1	2	5
f (x)	2	3	12	147

Using Lagrange's formula, the value of f (3) is

(1) 30 (2) 35

(3) 25 (4) 20

64. The value of function f (x) at 5 discrete points are given below :

x	0	0.1	0.2	0.3	0.4
f (x)	0	10	40	90	160

Using Trapezoidal rule with step size of 0.1, the value of $\int_0^{0.4} f(x)\,dx$ is

(1) 10.8 (2) 13.4

(3) 18.7 (4) 22.0

65. If $y' = x + y$, $y(0) = 1$, $y_1(x) = 1 + x + \frac{x^2}{2}$, then by Picard's method, the value of $y_2(x)$ is :

(1) $1 + x + x^2 + \frac{x^3}{6}$ (2) $1 - x + x^2 + \frac{x^3}{6}$

(3) $1 + x - x^2 + \frac{x^3}{6}$ (4) $1 + x + x^2 - \frac{x^3}{6}$

Question No.	Questions
66.	$I = \int_{x_1}^{x_2} F(y, y')\,dx$ whose ends are fixed is stationary if y satisfies the equation (1) $\frac{\partial F}{\partial y'}$ = constant (2) $F - y' \frac{\partial F}{\partial y'}$ = constant (3) $F - y \frac{\partial F}{\partial y'}$ = constant (4) $F' - y \frac{\partial F}{\partial y'}$ = constant
67.	If $J[y] = \int_1^2 (y'^2 + 2yy' + y^2)\,dx$, y (1) = 1 and y (2) is arbitrary, then the extremal is (1) e^{x-1} (2) e^{x+1} (3) e^{1-x} (4) e^{-x-1}
68.	The extremal of $\int_1^2 \frac{\dot{x}^2}{t^3}\,dt$; x (1) = 3, x (2) = 18 $\left(\text{where } \dot{x} \equiv \frac{dx}{dt}\right)$ using Lagrange's equation is given by which of the following ? (1) $x = t^4 + 2$ (2) $x = \frac{15}{7} t^3 + \frac{6}{7}$ (3) $x = 5t^2 - 2$ (4) $x = 5t^3 + 3$
69.	The kernel sin (x + t) is (1) separable kernel (2) difference kernel (3) adjoint kernel (4) none of these

Question No.	Questions
70.	The solution to the integral equation $\phi(x) = x + \int_0^x \sin(x-\xi)\,\phi(\xi)\,d\xi$ is given by (1) $x^2 + \frac{x^3}{3}$ (2) $x - \frac{x^3}{3!}$ (3) $x + \frac{x^3}{3!}$ (4) $x^2 - \frac{x^3}{3!}$
71.	The homogeneous integral equation $\phi(x) - \lambda \int_0^1 (3x-2)\,t\,\phi(t)\,dt = 0$, has (1) One characteristic number (2) Three characteristic numbers (3) Two characteristic numbers (4) No characteristic number
72.	Let S be a mechanical system with Lagrangian $L(q_j, \dot{q}_j, t), j = 1, 2, \ldots\ldots, n$ and generalized coordinates. Then the Lagrange equations of motion for S (1) constitute a set of n first order ODEs. (2) can be transformed to the Hamilton form using Legendre transform. (3) are equivalent to a set on n first order ODEs when expressed in terms of Hamiltonian functions. (4) is a set of 2n second order ODEs.

Question No.	Questions
73.	Lagrange's equations for a Holonomic dynamical system specified by n-generalized coordinates q_j ($j = 1, 2, 3 \ldots n$) having T as the K.E. of system at time t and Q_j the generalized forces are (1) $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right)+\frac{\partial T}{\partial q_j}=Q_j$ (2) $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right)-\frac{\partial T}{\partial q_j}=Q_j$ (3) $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right)-\frac{\partial T}{\partial q_j}=\dot{Q}_j$ (4) $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right)+\frac{\partial T}{\partial q_j}=\dot{Q}_j$
74.	Let q_i and $\dot{q}_i$ respectively are the generalized coordinates and velocity of a mechanical system and p_i are its generalized momenta. If H is the Hamiltonian of the system, then Hamilton's equations of motion are (1) $\dot{q}_i=\frac{\partial H}{\partial p_i}$, $\dot{p}_i=\frac{\partial H}{\partial q_i}$ (2) $\dot{q}_i=\frac{\partial H}{\partial p_i}$, $\dot{p}_i=-\frac{\partial H}{\partial q_i}$ (3) $\dot{q}_i=-\frac{\partial H}{\partial p_i}$, $\dot{p}_i=\frac{\partial H}{\partial q_i}$ (4) $\dot{q}_i=-\frac{\partial H}{\partial p_i}$, $\dot{p}_i=-\frac{\partial H}{\partial q_i}$
75.	Hamiltonian H is defined as (1) $H=\sum p_i \dot{q}_i - L$ (2) $H=\sum \dot{p}_i q_i - L$ (3) $H=\sum \dot{p}_i q_i + L$ (4) $H=\sum \dot{p}_i \dot{q}_i - L$
76.	What is the probability to get two aces in succession (with replacement) from a deck of 52 cards ? (1) 1/52 (2) 1/169 (3) 2/159 (4) 2/169

Question No.	Questions
77.	There are two boxes. Box 1 contains 2 red balls and 4 green balls. Box 2 contains 4 red balls and 2 green balls. A box is selected at random and a ball is chosen randomly from the selected box. If the ball turns out to be red, what is the probability that Box 1 had been selected ? (1) 1/2 (2) 1/6 (3) 2/3 (4) 1/3
78.	Suppose you have a coin with probability $\frac{3}{4}$ of getting a Head. You toss the coin twice independently. Let $\Omega = \{(H, H), (H, T), (T, H), (T, T)\}$ be the sample space. Then it is possible to have an event $E \subseteq \Omega$ such that (1) $P(E) = 1/3$ (2) $P(E) = 1/9$ (3) $P(E) = 1/4$ (4) $P(E) = 7/8$
79.	A random variable X has a probability distribution as follows : r: 0, 1, 2, 3 P (X = r): 2k, 3k, 13k, 2k Then the probability that P (X < 2) is equal to (1) 0.90 (2) 0.25 (3) 0.65 (4) 0.15
80.	Find the value of λ such that the function f (x) is a valid probability density function where $f(x) = \begin{cases} \lambda (x-1)(2-x) & \text{for } 1 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$ (1) 1 (2) 5 (3) 6 (4) 7

Table for Question 79:

r	0	1	2	3
P (X = r)	2k	3k	13k	2k

Question No.	Questions
81.	A continuous random variable X has a probability density function $f(x) = e^{-x}, 0 < x < \infty$. Then $P\{X > 1\}$ is (1) 0.368 (2) 0.5 (3) 0.632 (4) 1.0
82.	Let X and Y be two random variables having the joint probability density function $f(x, y) = \begin{cases} 2 & \text{if } 0 < x < y < 1 \\ 0 & \text{otherwise.} \end{cases}$ Then the conditional probability $P\left(X \leq \frac{2}{3} \middle\vert Y = \frac{3}{4}\right)$ is equal to (1) 5/9 (2) 2/3 (3) 7/9 (4) 8/9
83.	The variance of a random variable X is given by (1) $E[X - E(X)]^2$ (2) $[E(X)]^2 - E(X)^2$ (3) $E(X)^2 + (E(X))^2$ (4) None of these
84.	Standard Normal Variate has (1) Mean = 0 and variance = 1 (2) Mean = 1 and variance = 0 (3) Mean = 1 and variance = 1 (4) None of these
85.	When $n \to \infty$, the Binomial distribution can be approximated as (1) Bernoulli distribution (2) Uniform distribution (3) Poisson distribution (4) None of these

Question No.	Questions
86.	The variance of Poisson distribution is given by (1) $\sigma^2 = \lambda$ (2) $\sigma^2 = \frac{1}{\lambda}$ (3) $\sigma^2 = \frac{1}{\lambda^2}$ (4) None of these
87.	The first moment about origin is known as (1) Mean (2) Variance (3) Standard deviation (4) None of these
88.	In a hypothesis-testing problem, which of the following is not required in order to compute the p-value ? (1) Value of the test statistic (2) Distribution of the test statistic under the null hypothesis (3) The level of significance (4) Whether the test is one-sided or two-sided
89.	In testing H : $\mu = 100$ against A : $\mu \neq 100$ at the 10% level of significance, H is rejected if (1) 100 is contained in the 90% confidence interval (2) The value of the test statistic is in the acceptance region (3) The p-value is less than 0.10 (4) The p-value is greater than 0.10

Question No.	Questions
90.	In the context of testing of statistical hypothesis, which one of the following statements is true ? (1) When testing a simple hypothesis H_0 against an alternative simple hypothesis H_1, the likelihood ratio principle leads to the most powerful test. (2) When testing a simple hypothesis H_0 against an alternative simple hypothesis H_1, P [rejecting $H_0 \mid H_0$ is true] + P [accepting $H_0 \mid H_1$ is true] = 1. (3) For testing a simple hypothesis H_0 against an alternative simple hypothesis H_1, randomized test is used to achieve the desired level of the power of the test. (4) UMP test for testing a simple hypothesis H_0 against an alternative simple hypothesis H_1, always exist.
91.	A box contains N tickets which are numbered 1, 2, ..., N. Then value of N is however, unknown. A simple random sample of n tickets is drawn without replacement from the box. Let $X_1, X_2, ..., X_n$ be numbers on the tickets obtained in the 1^{st}, 2^{nd}, ..., n^{th} draws respectively. Here $\overline{X} = \frac{1}{N}(X_1 + X_2 + ... + X_n)$. Which of the following is an unbiased estimator of N ? (1) $2\overline{X} - 1$ (2) $2\overline{X} + 1$ (3) $2\overline{X} + \frac{1}{2}$ (4) $2\overline{X} - \frac{1}{2}$
92.	Let $X_1 \sim N(0, 1)$ and let $X_2 = \begin{cases} -X_1, & -2 \le X_1 \le 2 \\ X_1, & \text{otherwise.} \end{cases}$ Then identify the correct statement. (1) corr $(X_1, X_2) = 1$ (2) X_2 does not have N (0, 1) distribution. (3) (X_1, X_2) has a bivariate normal distribution. (4) (X_1, X_2) does not have a bivariate normal distribution.

Question No.	Questions
93.	Let $X_1, X_2, ..., X_n$ be a random sample of size n from a p-variate Normal distribution with mean μ and positive definite covariance matrix Σ. Choose the correct statement (1) $(X_1-\mu)' \sum^{-1} (X_1-\mu)$ has chi-suare distribution with 1 d.f. (2) $\overline{X}\,\overline{X}'$ has Wishart distribution with p d.f. (3) $\sum_{i=1}^{n} (X_i-\mu)(X_i-\mu)'$ has Wishart distribution with n d.f. (4) $X_1 + X_2$ and $X_1 - X_2$ are independently distributed.
94.	In which of the following distributions, mean $\geq$ variance (1) Poisson distribution (2) Negative binomial distribution (3) Normal distribution (4) Binomial distribution
95.	Let $X_1, X_2, ...$ be i.i.d. standard normal random variables and let $T_n = \frac{X_1^2 + ... + X_n^2}{n}$. Then (1) The limiting distribution of $T_n - 1$ is χ^2 with 1 degree of freedom. (2) The limiting distribution of $\frac{T_n - 1}{\sqrt{n}}$ is normal with mean 0 and variance 2. (3) The limiting distribution of $\sqrt{n}\,(T_n - 1)$ is χ^2 with 1 degree of freedom. (4) The limiting distribution of $\sqrt{n}\,(T_n - 1)$ is normal with mean 0 and variance 2.

Question No.	Questions
96.	Suppose the cumulative distribution function of failure time T of a component is $1 - \exp(-ct^{\alpha}),\ t > 0,\ \alpha > 1,\ c > 0.$ Then the hazard rate of $\lambda(t)$ is (1) constant. (2) non-constant monotonically increasing in t. (3) non-constant monotonically decreasing in t. (4) not a monotone function in t.
97.	Consider the following linear programming problem Maximize $z = 3x_1 + 2x_2$ subject to $x_1 + x_2 \geq 1$; $x_1 + x_2 \leq 5$; $2x_1 + 3x_2 \leq 6$; $-2x_1 + 3x_2 \leq 6$ The problem has (1) an unbounded solution (2) exactly one optimal solution (3) more than one optimal solution (4) no feasible solutions
98.	Let $\{X_n : n \geq 0\}$ be a Markov chain on a finite state space S with stationary transition probability matrix. Suppose that the chain is not irreducible. Then the Markov chain : (1) admits infinitely many stationary distributions (2) admits a unique stationary distribution (3) may not admit any stationary distribution (4) cannot admit exactly two stationary distributions

Question No.	Questions
99.	Men arrive in a queue according to a Poisson process with rate λ_1 and women arrive in the same queue according to another Poisson process with rate λ_2. The arrivals of mean and women are independent. The probability that the first arrival in the queue is a man is : (1) $\frac{\lambda_1}{\lambda_1+\lambda_2}$ (2) $\frac{\lambda_2}{\lambda_1+\lambda_2}$ (3) $\frac{\lambda_1}{\lambda_2}$ (4) $\frac{\lambda_2}{\lambda_1}$
100.	Let X (t) be the number of customers in an M/M/1 queuing system with arrival rate $\lambda > 0$ and service rate $\mu > 0$. The process X (t) is a (1) Poisson process with rate $\lambda - \mu$. (2) pure birth process with birth rate $\lambda - \mu$. (3) birth and death process with birth rate λ and death rate μ. (4) birth and death process with birth rate $1/\lambda$ and death rate $1/\mu$.

Question No.	Questions
1.	Let X and Y be metric spaces, and $f : X \rightarrow Y$ a function then which of the following is true (1) f is continuous ; (2) for every open set U in Y, f^{-1} (U) is open in X (3) for every closed set C in Y, f^{-1} (C) is closed in X (4) All the above
2.	The metric space (R, d), where d is a usual metric, is (1) compact (2) disconnected (3) connected but not compact (4) compact and connected
3.	Let (X, d) be a metric space, then for all $x, y, z \in X$ (1) $d(x, y) \leq d(x, z) + d(z, y)$ (2) $d(x, y) \geq d(x, z) + d(z, y)$ (3) $d(x, y) \leq 0$ (4) None of these
4.	A normed linear space X is complete iff (1) Every convergent series in X is absolutely convergent (2) Every convergent series in X is convergent (3) Every convergent series in X is uniformly convergent (4) Every absolutely convergent series in X is convergent
5.	If W_1, W_2 are two subspaces of a finite dimension vector space V(F), then (1) $\dim (W_1 + W_2) = \dim (W_1 \cup W_2)$ (2) $\dim (W_1 + W_2) = \dim W_1 + \dim W_2$ (3) $\dim (W_1 + W_2) = \dim W_1 + \dim W_2 - \dim (W_1 \cap W_2)$ (4) $\dim (W_1 + W_2) = \dim W_1 + \dim W_2 + \dim (W_1 \cap W_2)$

Question No.	Questions
6.	The co-ordinates of vector (1, 1, 1) relative to basis (1, 1, 2), (2, 2, 1), (1, 2, 2) is (1) (1/3, 0, 1/3) (2) (1/3, 1/3, 0) (3) (0, 1/3, 1/3) (4) None of these
7.	Let T : $R^n \to R^n$ be a linear transformation. Which of the following statements implies that T is bijective ? (1) Nullity (T) = n (2) Rank (T) = Nullity (T) = n (3) Rank (T) + Nullity (T) = n (4) Rank (T) – Nullity (T) = n
8.	Let A, B be n × n real matrices. Which of the following statements is correct ? (1) rank (A + B) = rank(A) + rank(B) (2) rank (A + B) ≤ rank(A) + rank(B) (3) rank (A + B) = min {rank(A), rank(B)} (4) rank (A + B) = max {rank (A), rank(B)}
9.	Let A and B be real invertible matrices such that AB = – BA. Then (1) Trace (A) = 1, Trace (B) = 0 (2) Trace (A) = Trace (B) = 1 (3) Trace (A) = 0, Trace (B) = 1 (4) Trace (A) = Trace (B) = 0
10.	Consider the matrix $A(x) = \begin{bmatrix} 1+x^2 & 7 & 11 \\ 3x & 2x & 4 \\ 8x & 17 & 13 \end{bmatrix}$; $x \in R$. Then (1) A (x) has eigenvalue 0 for some $x \in R$ (2) 0 is not an eigenvalue of A(x) for any $x \in R$ (3) A (x) has eigenvalue 0 for all $x \in R$ (4) A (x) is invertible for every $x \in R$

Question No.	Questions
11.	A box contains N tickets which are numbered 1, 2, ..., N. Then value of N is however, unknown. A simple random sample of n tickets is drawn without replacement from the box. Let $X_1, X_2, ..., X_n$ be numbers on the tickets obtained in the 1st, 2nd, ..., nth draws respectively. Here $\overline{X} = \frac{1}{N}(X_1 + X_2 + ... + X_n)$. Which of the following is an unbiased estimator of N ? (1) $2\overline{X} - 1$ (2) $2\overline{X} + 1$ (3) $2\overline{X} + \frac{1}{2}$ (4) $2\overline{X} - \frac{1}{2}$
12.	Let $X_1 \sim N(0, 1)$ and let $X_2 = \begin{cases} -X_1, & -2 \le X_1 \le 2 \\ X_1, & \text{otherwise.} \end{cases}$ Then identify the correct statement. (1) corr $(X_1, X_2) = 1$ (2) X_2 does not have N (0, 1) distribution. (3) (X_1, X_2) has a bivariate normal distribution. (4) (X_1, X_2) does not have a bivariate normal distribution.
13.	Let $X_1, X_2, ..., X_n$ be a random sample of size n from a p-variate Normal distribution with mean μ and positive definite covariance matrix Σ. Choose the correct statement (1) $(X_1 - \mu)' \sum^{-1} (X_1 - \mu)$ has chi-suare distribution with 1 d.f. (2) $\overline{X}\,\overline{X}'$ has Wishart distribution with p d.f. (3) $\sum_{i=1}^{n} (X_i - \mu)(X_i - \mu)'$ has Wishart distribution with n d.f. (4) $X_1 + X_2$ and $X_1 - X_2$ are independently distributed.

Question No.	Questions
14.	In which of the following distributions, mean ≥ variance (1) Poisson distribution (2) Negative binomial distribution (3) Normal distribution (4) Binomial distribution
15.	Let $X_1, X_2, \ldots$ be i.i.d. standard normal random variables and let $T_n = \frac{X_1^2 + \ldots + X_n^2}{n}$. Then (1) The limiting distribution of $T_n - 1$ is χ^2 with 1 degree of freedom. (2) The limiting distribution of $\frac{T_n - 1}{\sqrt{n}}$ is normal with mean 0 and variance 2. (3) The limiting distribution of $\sqrt{n}\,(T_n - 1)$ is χ^2 with 1 degree of freedom. (4) The limiting distribution of $\sqrt{n}\,(T_n - 1)$ is normal with mean 0 and variance 2.
16.	Suppose the cumulative distribution function of failure time T of a component is $1 - \exp(-ct^{\alpha}),\ \ t > 0,\ \alpha > 1,\ c > 0.$ Then the hazard rate of $\lambda(t)$ is (1) constant. (2) non-constant monotonically increasing in t. (3) non-constant monotonically decreasing in t. (4) not a monotone function in t.

Question No.	Questions
17.	Consider the following linear programming problem Maximize $z = 3x_1 + 2x_2$ subject to $x_1 + x_2 \geq 1$; $x_1 + x_2 \leq 5$; $2x_1 + 3x_2 \leq 6$; $-2x_1 + 3x_2 \leq 6$ The problem has (1) an unbounded solution (2) exactly one optimal solution (3) more than one optimal solution (4) no feasible solutions
18.	Let $\{X_n : n \geq 0\}$ be a Markov chain on a finite state space S with stationary transition probability matrix. Suppose that the chain is not irreducible. Then the Markov chain : (1) admits infinitely many stationary distributions (2) admits a unique stationary distribution (3) may not admit any stationary distribution (4) cannot admit exactly two stationary distributions
19.	Men arrive in a queue according to a Poisson process with rate λ_1 and women arrive in the same queue according to another Poisson process with rate λ_2. The arrivals of mean and women are independent. The probability that the first arrival in the queue is a man is : (1) $\frac{\lambda_1}{\lambda_1 + \lambda_2}$ (2) $\frac{\lambda_2}{\lambda_1 + \lambda_2}$ (3) $\frac{\lambda_1}{\lambda_2}$ (4) $\frac{\lambda_2}{\lambda_1}$

Question No.	Questions
20.	Let X (t) be the number of customers in an M/M/1 queuing system with arrival rate $\lambda > 0$ and service rate $\mu > 0$. The process X (t) is a (1) Poisson process with rate $\lambda - \mu$. (2) pure birth process with birth rate $\lambda - \mu$. (3) birth and death process with birth rate λ and death rate μ. (4) birth and death process with birth rate $1/\lambda$ and death rate $1/\mu$.
21.	The homogeneous integral equation $\phi(x) - \lambda \int_0^1 (3x-2)\, t\, \phi(t)\, dt = 0$, has (1) One characteristic number (2) Three characteristic numbers (3) Two characteristic numbers (4) No characteristic number
22.	Let S be a mechanical system with Lagrangian $L(q_j, \dot{q}_j, t)$, $j = 1, 2, \ldots\ldots, n$ and generalized coordinates. Then the Lagrange equations of motion for S (1) constitute a set of n first order ODEs. (2) can be transformed to the Hamilton form using Legendre transform. (3) are equivalent to a set on n first order ODEs when expressed in terms of Hamiltonian functions. (4) is a set of 2n second order ODEs.

Question No.	Questions
23.	Lagrange's equations for a Holonomic dynamical system specified by n-generalized coordinates q_j (j = 1, 2, 3 ... n) having T as the K.E. of system at time t and Q_j the generalized forces are (1) $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right)+\frac{\partial T}{\partial q_j}=Q_j$ (2) $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right)-\frac{\partial T}{\partial q_j}=Q_j$ (3) $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right)-\frac{\partial T}{\partial q_j}=\dot{Q}_j$ (4) $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right)+\frac{\partial T}{\partial q_j}=\dot{Q}_j$
24.	Let q_i and $\dot{q}_i$ respectively are the generalized coordinates and velocity of a mechanical system and p_i are its generalized momenta. If H is the Hamiltonian of the system, then Hamilton's equations of motion are (1) $\dot{q}_i=\frac{\partial H}{\partial p_i}$, $\dot{p}_i=\frac{\partial H}{\partial q_i}$ (2) $\dot{q}_i=\frac{\partial H}{\partial p_i}$, $\dot{p}_i=-\frac{\partial H}{\partial q_i}$ (3) $\dot{q}_i=-\frac{\partial H}{\partial p_i}$, $\dot{p}_i=\frac{\partial H}{\partial q_i}$ (4) $\dot{q}_i=-\frac{\partial H}{\partial p_i}$, $\dot{p}_i=-\frac{\partial H}{\partial q_i}$
25.	Hamiltonian H is defined as (1) $H=\sum p_i \dot{q}_i - L$ (2) $H=\sum \dot{p}_i q_i - L$ (3) $H=\sum \dot{p}_i q_i + L$ (4) $H=\sum \dot{p}_i \dot{q}_i - L$
26.	What is the probability to get two aces in succession (with replacement) from a deck of 52 cards ? (1) 1/52 (2) 1/169 (3) 2/159 (4) 2/169

Question No.	Questions
27.	There are two boxes. Box 1 contains 2 red balls and 4 green balls. Box 2 contains 4 red balls and 2 green balls. A box is selected at random and a ball is chosen randomly from the selected box. If the ball turns out to be red, what is the probability that Box 1 had been selected ? (1) 1/2 (2) 1/6 (3) 2/3 (4) 1/3
28.	Suppose you have a coin with probability $\frac{3}{4}$ of getting a Head. You toss the coin twice independently. Let $\Omega = \{(H, H), (H, T), (T, H), (T, T)\}$ be the sample space. Then it is possible to have an event $E \subseteq \Omega$ such that (1) $P(E) = 1/3$ (2) $P(E) = 1/9$ (3) $P(E) = 1/4$ (4) $P(E) = 7/8$
29.	A random variable X has a probability distribution as follows : <table><tr><td>r</td><td>0</td><td>1</td><td>2</td><td>3</td></tr><tr><td>P (X = r)</td><td>2k</td><td>3k</td><td>13k</td><td>2k</td></tr></table> Then the probability that $P(X < 2)$ is equal to (1) 0.90 (2) 0.25 (3) 0.65 (4) 0.15
30.	Find the value of λ such that the function f (x) is a valid probability density function where $f(x) = \begin{cases} \lambda (x-1)(2-x) & \text{for } 1 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$ (1) 1 (2) 5 (3) 6 (4) 7

Question No.	Questions
31.	Consider the initial value problem (IVP) $$\frac{dy}{dx} = y^2,\ y(0) = 1,\ (x, y) \in R \times R.$$ Then there exists a unique solution of the IVP on (1) $(-\infty, \infty)$ (2) $(-\infty, 1)$ (3) $(-2, 2)$ (4) $(-1, \infty)$
32.	The solution to the initial value problem $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 5y = 3e^{-t} \sin t$, $y(0) = 0$ and $y'(0) = 3$, is (1) $y(t) = e^t (\sin t + \sin 2t)$ (2) $y(t) = e^{-t} (\sin t + \sin 2t)$ (3) $y(t) = 3e^t \sin t$ (4) $y(t) = 3e^{-t} \sin t$
33.	Consider the differential equation $(x-1)y'' + xy' + \frac{1}{x} y = 0$. Then (1) $x = 1$ is the only singular point (2) $x = 0$ is the only singular point (3) both $x = 0$ and $x = 1$ are singular points (4) neither $x = 0$ nor $x = 1$ are singular points
34.	Let f and g be real linearly independent solutions of $$\frac{d}{dx}\left[\frac{dy}{dx} P(x)\right] + Q(x)\, y = 0$$ on the interval $a \le x \le b$. Then (1) between any two consecutive zeros of f, there is precisely one zero of g. (2) between any two consecutive zeros of f, there is no zero of g. (3) between any two consecutive zeros of f, there is infinite zeros of g. (4) None of these

Question No.	Questions
35.	The eigen values of a Sturm-Liouville BVP are (1) Always positive (2) Always negative (3) Always real (4) Always in the pair of complex conjugate
36.	The Charpit's equations for the PDE $up^2 + q^2 + x + y = 0$, $p = \frac{\partial u}{\partial x}$, $q = \frac{\partial u}{\partial y}$ are given by (1) $\frac{dx}{-1-p^3} = \frac{dy}{-1-qp^2} = \frac{du}{2p^2u+2q^2} = \frac{dp}{2pu} = \frac{dq}{2q}$ (2) $\frac{dx}{2pu} = \frac{dy}{2q} = \frac{du}{2p^2u+2q^2} = \frac{dp}{-1-p^3} = \frac{dq}{-1-qp^2}$ (3) $\frac{dx}{up^2} = \frac{dy}{q^2} = \frac{du}{0} = \frac{dp}{x} = \frac{dq}{y}$ (4) $\frac{dx}{2q} = \frac{dy}{2pu} = \frac{du}{x+y} = \frac{dp}{p^2} = \frac{dq}{qp^2}$
37.	The partial differential equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + u$ can be transformed to $\frac{\partial v}{\partial t} = \frac{\partial^2 v}{\partial x^2}$ for (1) $v = e^{-t} u$. (2) $v = e^{t} u$. (3) $v = tu$. (4) $v = -tu$.

Question No.	Questions
38.	The PDE $x\frac{\partial^2 u}{\partial x^2}+y\frac{\partial^2 u}{\partial y^2}=0$ is (1) elliptic for $x<0, y>0$ (2) hyperbolic for $x>0, y<0$ (3) elliptic for $x>0, y<0$ (4) hyperbolic for $x>0, y>0$
39.	Solution of $\frac{\partial^2 z}{\partial x^2}-\frac{\partial^2 z}{\partial x\,\partial y}-6\frac{\partial^2 z}{\partial y^2}=0$ is given by (1) $z=f_1(y+3x)+f_2(y+2x)$ (2) $z=f_1(y+3x)+f_2(y-2x)$ (3) $z=f_1(y-2x)+f_2(y+2x)$ (4) None of these
40.	Given Wave equation $\frac{\partial z}{\partial y}=c^2\frac{\partial^2 z}{\partial x^2}$. By separation of variable, let $z(x, y)=X(x)\,Y(y)$ be a solution. Substituting it in $\frac{\partial z}{\partial y}=c^2\frac{\partial^2 z}{\partial x^2}$ one have $\frac{d^2X}{dx^2}+kX=0$ and $\frac{dY}{dy}-kc^2Y=0$. If $k=-p^2$, p is real, then the solution is (c's are constants) (1) $z(x,y)=(c_1\cos px)\,c_2\,e^{-c^2p^2y}$ (2) $z(x,y)=(c_1\cos px+c_2\sin px)\,c_1\,e^{-c^2p^2y}$ (3) $z(x,y)=e^{-c^2p^2y}$ (4) $z(x,y)=(c_2\cos px)\,c_1\,e^{-c^2p^2y}$

Question No.	Questions
41.	The statement, if f is entire and bounded for all $z \in C$, then f is constant, refers to : (1) Morera's theorem (2) Maximum modulus theorem (3) Liouville's theorem (4) Hurwitz theorem
42.	Let f(z) be analytic in a closed, connected domain, D then which of the following is not true (1) The extreme values of the modulus of the function must occur on the boundary (2) If $\|f(z)\|$ has an interior extrema, then the function is a constant (3) The extreme values of the modulus of the function may occur on interior point and f(z) may not be constant (4) If f(z) is non constant then $\|f(z)\|$ does not attains an extrema inside the boundary
43.	The function $f : C \to C$ defined by $f(z) = e^z + e^{-z}$ has (1) finitely many zeros (2) no zeros (3) only real zeros (4) has infinitely may zeros
44.	Consider the following complex function $f(z) = \dfrac{9}{(z-1)(z+2)^2}$. Which of the following is one of the residues of the above function ? (1) –1 (2) 9/16 (3) 2 (4) 9
45.	The bilinear transformation that maps the points z = ∞, i, 0 into the points w = 0, i, ∞ is (1) $w = -z$ (2) $w = z$ (3) $w = \dfrac{-1}{z}$ (4) $w = \dfrac{1}{z}$

Question No.	Questions
46.	The fundamental theorem of arithmetic states that (1) The factoring of any integer n > 1 into primes is not unique apart from the order of prime factors (2) The factoring of any integer n > 1 into primes is unique apart from the order of prime factors (3) There are infinitely many primes (4) The number of prime numbers is finite
47.	The last two digits of 7^{81} are (1) 07 (2) 17 (3) 37 (4) 47
48.	The congruence $35x \equiv 14 \pmod{21}$ has (1) 7 solutions (2) 6 solutions (3) 9 solutions (4) No solution
49.	If n is a positive integer such that the sum of all positive integers a satisfying $1 \leq a \leq n$ and GCD (a, n) = 1 is equal to 240n, then the number of summands, namely, $\phi(n)$, is (1) 120 (2) 124 (3) 240 (4) 480
50.	If gcd (m, n) = 1 where m > 2 and n > 2, then the integer mn has (1) no primitive roots (2) unique primitive root (3) infinite primitive roots (4) finite primitive roots

Question No.	Questions
51.	The Linear transformation $T : R^3 \to R^3$ corresponding to the matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ is (1) $T(x_1, x_2, x_3) = (x_1, 2x_2, 3x_3)$ (2) $T(x_1, x_2, x_3) = (x_1 + x_3, 2x_1 + x_2, x_2 + x_3)$ (3) $T(x_1, x_2, x_3) = (x_1, x_2, x_3)$ (4) None of these
52.	The matrix $\begin{bmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix}$ is (1) non-negative definite but not positive definite (2) positive definite (3) negative definite (4) neither negative definite nor positive definite
53.	Let $X = \begin{bmatrix} 2 & 0 & -3 \\ 3 & -1 & -3 \\ 0 & 0 & -1 \end{bmatrix}$. A matrix P such that $P^{-1}XP$ is a diagonal matrix, is (1) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ (2) $\begin{bmatrix} -1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ (3) $\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ (4) $\begin{bmatrix} -1 & -1 & 1 \\ 0 & -1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

Question No.	Questions
54.	The norm of x with respect to inner product space $\langle x, x\rangle$ is (1) $\|x\| = \langle x, x\rangle$ (2) $\|x\|^2 = \langle x, x\rangle$ (3) $\|x\| = \langle x, x\rangle^2$ (4) None of these
55.	Cayley-Hamilton theorem states that (1) Every square matrix satisfies its own characteristic equation (2) Every square matrix does not satisfy its own characteristic equation (3) Every rectangular matrix satisfies its own characteristic equation (4) None of these
56.	If $\|z\| = \|z-1\|$ then (1) Re (z) = 1 (2) Re (z) = 1/2 (3) Im (z) = 1 (4) Im (z) = 1/2
57.	The power series $\sum_{n=0}^{\infty} 3^{-n} (z-1)^{2n}$ converges if (1) $\|z\| \leq 3$ (2) $\|z\| < \sqrt{3}$ (3) $\|z-1\| < \sqrt{3}$ (4) $\|z-1\| \leq \sqrt{3}$
58.	An analytic function of a complex variable z = x + iy is expressed as $f(z) = u(x, y) + iv(x, y)$, where $i = \sqrt{-1}$. If $u(x, y) = 2xy$, then $v(x, y)$ must be (1) $x^2 + y^2$ + constant (2) $x^2 - y^2$ + constant (3) $-x^2 + y^2$ + constant (4) $-x^2 - y^2$ + constant
59.	$\int_{\|z\|=2} \frac{2z}{z^2+2} dz =$ (1) 0 (2) $-2\pi i$ (3) $4\pi i$ (4) 1

Question No.	Questions
60.	The value of $\int_C \frac{\sin z}{4z+\pi} dz$ where C : $\|z\| = 1$ is a positively oriented contour. (1) 0 (2) $\frac{-\sqrt{2}\,\pi i}{4}$ (3) $\frac{-\sqrt{2}\,i}{4}$ (4) $\frac{-\pi i}{4}$
61.	If p is a prime, then any group G of order 2p has (1) a normal subgroup of order p (2) a normal subgroup of order 2p (3) a normal subgroup of order p^2 (4) None of these
62.	Let G be simple group of order 60. Then (1) G has six Sylow-5 subgroups (2) G has four Sylow-3 subgroups (3) G has a cyclic subgroup of order 6 (4) G has a unique element of order 2
63.	Let R be a Euclidean domain such that R is not a field. Then the polynomial ring R[X] is always (1) a Euclidean domain (2) a principal ideal domain but not a Euclidean domain (3) a unique factorization domain but not a principal ideal domain (4) not a unique factorization domain

Question No.	Questions
64.	Let $p(x) = 9x^5 + 10x^3 + 5x + 15$ and $q(x) = x^3 - x^2 - x - 2$ be two polynomials in Q[x]. Then over Q, (1) p(x) and q(x) are both irreducible (2) p(x) is reducible but q(x) is irreducible (3) p(x) is irreducible but q(x) is reducible (4) p(x) and q(x) are both reducible
65.	Find the degree of the field extension $Q\left(\sqrt{2}, \sqrt[4]{2}, \sqrt[8]{2}\right)$ over Q. (1) 4 (2) 8 (3) 14 (4) 32
66.	Let F be a finite field and let K/F be a field extension of degree 6. Then the Galois group of K/F is isomorphic to (1) the cyclic group of order 6 (2) the permutation group of {1, 2, 3} (3) the permutation group on {1, 2, 3, 4, 5, 6} (4) the permutation group on {1}
67.	Let X be a topological space and A be a subset of X, then X is separable if (1) A is countable and $\overline{A} = X$ (2) $\overline{A}$ is countable (3) A is uncountable (4) None of these
68.	Which of the following spaces is not separable ? (1) R with the trivial topology (2) The Cantor set as a subspace of R (3) R with the discrete topology (4) None of these

Question No.	Questions
69.	Which of the following is true ? (1) Let X be compact and $f : X \to R$ be locally bounded. Then f is not bounded. (2) Closed subspaces of compact spaces are compact (3) Closed subspaces of compact spaces may not be compact (4) Continuous images of compact spaces may not be compact
70.	Let X and Y be two topological spaces and let $f : X \to Y$ be a continuous function. Then (1) f (K) is connected if $K \subset X$ is connected (2) f^{-1} (K) is connected if $K \subset Y$ is connected (3) f^{-1} (K) is compact if $K \subset Y$ is compact (4) None of these
71.	The rate of convergence is faster for (1) Regula-Falsi method (2) Bisection method (3) Newton-Raphson method (4) Cannot say
72.	As soon as a new value of a variable is found by iteration, it is used immediately in the following equations, this method is called (1) Gauss-Jordan method (2) Gauss-Seidal method (3) Jacobi's method (4) Relaxation method
73.	The value of function f (x) at 4 discrete points are givne below : x: 0, 1, 2, 5 f (x): 2, 3, 12, 147 Using Lagrange's formula, the value of f (3) is (1) 30 (2) 35 (3) 25 (4) 20

Table for Question 73:

x	0	1	2	5
f (x)	2	3	12	147

Question No.	Questions
74.	The value of function f (x) at 5 discrete points are given below : x: 0, 0.1, 0.2, 0.3, 0.4 f(x): 0, 10, 40, 90, 160 Using Trapezoidal rule with step size of 0.1, the value of $\int_0^{0.4} f(x)\,dx$ is (1) 10.8 (2) 13.4 (3) 18.7 (4) 22.0
75.	If $y' = x + y$, $y(0) = 1$, $y_1(x) = 1 + x + \frac{x^2}{2}$, then by Picard's method, the value of $y_2(x)$ is : (1) $1 + x + x^2 + \frac{x^3}{6}$ (2) $1 - x + x^2 + \frac{x^3}{6}$ (3) $1 + x - x^2 + \frac{x^3}{6}$ (4) $1 + x + x^2 - \frac{x^3}{6}$
76.	$I = \int_{x_1}^{x_2} F(y, y')\,dx$ whose ends are fixed is stationary if y satisfies the equation (1) $\frac{\partial F}{\partial y'} = \text{constant}$ (2) $F - y' \frac{\partial F}{\partial y'} = \text{constant}$ (3) $F - y \frac{\partial F}{\partial y'} = \text{constant}$ (4) $F' - y \frac{\partial F}{\partial y'} = \text{constant}$

Question No.	Questions
77.	If J [y] = $\int_1^2 (y'^2 + 2yy' + y^2)\,dx$, y (1) = 1 and y (2) is arbitrary, then the extremal is (1) e^{x-1} (2) e^{x+1} (3) e^{1-x} (4) e^{-x-1}
78.	The extremal of $\int_1^2 \frac{\dot{x}^2}{t^3}\,dt$; x (1) = 3, x (2) = 18 $\left(\text{where } \dot{x} \equiv \frac{dx}{dt}\right)$ using Lagrange's equation is given by which of the following ? (1) $x = t^4 + 2$ (2) $x = \frac{15}{7} t^3 + \frac{6}{7}$ (3) $x = 5t^2 - 2$ (4) $x = 5t^3 + 3$
79.	The kernel sin (x + t) is (1) separable kernel (2) difference kernel (3) adjoint kernel (4) none of these
80.	The solution to the integral equation $\phi(x) = x + \int_0^x \sin(x - \xi)\,\phi(\xi)\,d\xi$ is given by (1) $x^2 + \frac{x^3}{3}$ (2) $x - \frac{x^3}{3!}$ (3) $x + \frac{x^3}{3!}$ (4) $x^2 - \frac{x^3}{3!}$

Question No.	Questions
81.	The non-empty set of real numbers which is bounded below has (1) supremum (2) infimum (3) upper bound (4) none of these
82.	The sequence $\{f_n\}$ where $f_n(x) = x^n$ is ________ convergent on [0, k], k < 1 (1) uniformly (2) pointwise (3) nowhere (4) none of these
83.	Every bounded sequence has at least one limit point. This represents (1) Archimedean Property (2) Heine-Borel theorem (3) Bolzano-Weierstress theorem (4) Denseness Property
84.	Which of the following is convergent ? (1) $\sum_{n=1}^{\infty} n^2 2^{-n}$ (2) $\sum_{n=1}^{\infty} n^{-2} 2^n$ (3) $\sum_{n=2}^{\infty} \frac{1}{n \log n}$ (4) $\sum_{n=1}^{\infty} \frac{1}{n \log(1+1/n)}$
85.	If a function f defined on [0, 1] as $f(x) = \begin{cases} i, \text{if } x \neq 1/2 \\ 0, \text{if } x = 1/2 \end{cases}$, then (1) f is not bounded (2) f is R-integrable (3) f is not R-integrable since f is not bounded (4) f is not R-integrable since lower and upper limits are unequal
86.	Let $f : R \rightarrow R$ be a monotone function. Then (1) f has no discontinuities (2) f has only finitely many discontinuities. (3) f can have at most countably many discontinuities (4) f can have uncountably many discontinuities

Question No.	Questions
87.	The length of an interval I is (1) Outer measure of an interval I (2) Less than outer measure of an interval I (3) Greater than outer measure of an interval I (4) Twice the outer measure of an interval I
88.	A set E is said to be Lebesgue measurable if for each set A (1) $m^*(A) = m^*(A \cap E) - m^*(A \cap E^c)$ (2) $m^*(A) = m^*(A \cap E^c) - m^*(A \cap E)$ (3) $m^*(A) = m^*(A \cup E) + m^*(A \cap E^c)$ (4) $m^*(A) = m^*(A \cup E) - m^*(A \cap E^c)$
89.	A non-negative measurable function f is integrable over the measurable set E if (1) $\int_E f = \infty$ (2) $\int_E f > \infty$ (3) $\int_E f < \infty$ (4) None of these
90.	If f is of bounded variation on [a, b] and $c \in (a, b)$. Then (1) f is of bounded variation on [a, c] and on [c, b] (2) f is not of bounded variation on [a, c] and on [c, b] (3) f is constant on [a, c] and on [c, b] (4) None of these
91.	A continuous random variable X has a probability density function $f(x) = e^{-x}, 0 < x < \infty$. Then P {X > 1} is (1) 0.368 (2) 0.5 (3) 0.632 (4) 1.0

Question No.	Questions
92.	Let X and Y be two random variables having the joint probability density function $f(x, y) = \begin{cases} 2 & \text{if } 0 < x < y < 1 \\ 0 & \text{otherwise.} \end{cases}$ Then the conditional probability $P\left(X \leq \frac{2}{3} \middle\| Y = \frac{3}{4}\right)$ is equal to (1) 5/9 (2) 2/3 (3) 7/9 (4) 8/9
93.	The variance of a random variable X is given by (1) $E[X - E(X)]^2$ (2) $[E(X)]^2 - E(X)^2$ (3) $E(X)^2 + (E(X))^2$ (4) None of these
94.	Standard Normal Variate has (1) Mean = 0 and variance = 1 (2) Mean = 1 and variance = 0 (3) Mean = 1 and variance = 1 (4) None of these
95.	When $n \to \infty$, the Binomial distribution can be approximated as (1) Bernoulli distribution (2) Uniform distribution (3) Poisson distribution (4) None of these
96.	The variance of Poisson distribution is given by (1) $\sigma^2 = \lambda$ (2) $\sigma^2 = \frac{1}{\lambda}$ (3) $\sigma^2 = \frac{1}{\lambda^2}$ (4) None of these

Question No.	Questions
97.	The first moment about origin is known as (1) Mean (2) Variance (3) Standard deviation (4) None of these
98.	In a hypothesis-testing problem, which of the following is not required in order to compute the p-value ? (1) Value of the test statistic (2) Distribution of the test statistic under the null hypothesis (3) The level of significance (4) Whether the test is one-sided or two-sided
99.	In testing H : $\mu = 100$ against A : $\mu \neq 100$ at the 10% level of significance, H is rejected if (1) 100 is contained in the 90% confidence interval (2) The value of the test statistic is in the acceptance region (3) The p-value is less than 0.10 (4) The p-value is greater than 0.10
100.	In the context of testing of statistical hypothesis, which one of the following statements is true ? (1) When testing a simple hypothesis H_0 against an alternative simple hypothesis H_1, the likelihood ratio principle leads to the most powerful test. (2) When testing a simple hypothesis H_0 against an alternative simple hypothesis H_1, P [rejecting $H_0 \mid H_0$ is true] + P [accepting $H_0 \mid H_1$ is true] = 1. (3) For testing a simple hypothesis H_0 against an alternative simple hypothesis H_1, randomized test is used to achieve the desired level of the power of the test. (4) UMP test for testing a simple hypothesis H_0 against an alternative simple hypothesis H_1, always exist.

Question No.	Questions
1.	If p is a prime, then any group G of order 2p has (1) a normal subgroup of order p (2) a normal subgroup of order 2p (3) a normal subgroup of order p^2 (4) None of these
2.	Let G be simple group of order 60. Then (1) G has six Sylow-5 subgroups (2) G has four Sylow-3 subgroups (3) G has a cyclic subgroup of order 6 (4) G has a unique element of order 2
3.	Let R be a Euclidean domain such that R is not a field. Then the polynomial ring R[X] is always (1) a Euclidean domain (2) a principal ideal domain but not a Euclidean domain (3) a unique factorization domain but not a principal ideal domain (4) not a unique factorization domain
4.	Let $p(x) = 9x^5 + 10x^3 + 5x + 15$ and $q(x) = x^3 - x^2 - x - 2$ be two polynomials in Q[x]. Then over Q, (1) p(x) and q(x) are both irreducible (2) p(x) is reducible but q(x) is irreducible (3) p(x) is irreducible but q(x) is reducible (4) p(x) and q(x) are both reducible
5.	Find the degree of the field extension $Q\left(\sqrt{2}, \sqrt[4]{2}, \sqrt[8]{2}\right)$ over Q. (1) 4 (2) 8 (3) 14 (4) 32

Question No.	Questions
6.	Let F be a finite field and let K/F be a field extension of degree 6. Then the Galois group of K/F is isomorphic to (1) the cyclic group of order 6 (2) the permutation group of {1, 2, 3} (3) the permutation group on {1, 2, 3, 4, 5, 6} (4) the permutation group on {1}
7.	Let X be a topological space and A be a subset of X, then X is separable if (1) A is countable and $\overline{A} = X$ (2) $\overline{A}$ is countable (3) A is uncountable (4) None of these
8.	Which of the following spaces is not separable ? (1) R with the trivial topology (2) The Cantor set as a subspace of R (3) R with the discrete topology (4) None of these
9.	Which of the following is true ? (1) Let X be compact and $f : X \rightarrow R$ be locally bounded. Then f is not bounded. (2) Closed subspaces of compact spaces are compact (3) Closed subspaces of compact spaces may not be compact (4) Continuous images of compact spaces may not be compact
10.	Let X and Y be two topological spaces and let $f : X \rightarrow Y$ be a continuous function. Then (1) $f(K)$ is connected if $K \subset X$ is connected (2) $f^{-1}(K)$ is connected if $K \subset Y$ is connected (3) $f^{-1}(K)$ is compact if $K \subset Y$ is compact (4) None of these

Question No.	Questions
11.	The Linear transformation $T : R^3 \to R^3$ corresponding to the matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ is (1) $T(x_1, x_2, x_3) = (x_1, 2x_2, 3x_3)$ (2) $T(x_1, x_2, x_3) = (x_1 + x_3, 2x_1 + x_2, x_2 + x_3)$ (3) $T(x_1, x_2, x_3) = (x_1, x_2, x_3)$ (4) None of these
12.	The matrix $\begin{bmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix}$ is (1) non-negative definite but not positive definite (2) positive definite (3) negative definite (4) neither negative definite nor positive definite
13.	Let $X = \begin{bmatrix} 2 & 0 & -3 \\ 3 & -1 & -3 \\ 0 & 0 & -1 \end{bmatrix}$. A matrix P such that $P^{-1} XP$ is a diagonal matrix, is (1) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ (2) $\begin{bmatrix} -1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ (3) $\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ (4) $\begin{bmatrix} -1 & -1 & 1 \\ 0 & -1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

Question No.	Questions
14.	The norm of x with respect to inner product space $\langle x, x\rangle$ is (1) $\lVert x\rVert = \langle x, x\rangle$ (2) $\lVert x\rVert^2 = \langle x, x\rangle$ (3) $\lVert x\rVert = \langle x, x\rangle^2$ (4) None of these
15.	Cayley-Hamilton theorem states that (1) Every square matrix satisfies its own characteristic equation (2) Every square matrix does not satisfy its own characteristic equation (3) Every rectangular matrix satisfies its own characteristic equation (4) None of these
16.	If $\lvert z\rvert = \lvert z-1\rvert$ then (1) $\text{Re}(z) = 1$ (2) $\text{Re}(z) = 1/2$ (3) $\text{Im}(z) = 1$ (4) $\text{Im}(z) = 1/2$
17.	The power series $\sum_{n=0}^{\infty} 3^{-n}(z-1)^{2n}$ converges if (1) $\lvert z\rvert \le 3$ (2) $\lvert z\rvert < \sqrt{3}$ (3) $\lvert z-1\rvert < \sqrt{3}$ (4) $\lvert z-1\rvert \le \sqrt{3}$
18.	An analytic function of a complex variable $z = x + iy$ is expressed as $f(z) = u(x, y) + iv(x, y)$, where $i = \sqrt{-1}$. If $u(x, y) = 2xy$, then $v(x, y)$ must be (1) $x^2 + y^2$ + constant (2) $x^2 - y^2$ + constant (3) $-x^2 + y^2$ + constant (4) $-x^2 - y^2$ + constant
19.	$\int_{\lvert z\rvert=2} \frac{2z}{z^2+2}\,dz =$ (1) 0 (2) $-2\pi i$ (3) $4\pi i$ (4) 1

Question No.	Questions
20.	The value of $\int_C \frac{\sin z}{4z+\pi} dz$ where C : $\vert z \vert = 1$ is a positively oriented contour. (1) 0 (2) $\frac{-\sqrt{2}\,\pi i}{4}$ (3) $\frac{-\sqrt{2}\,i}{4}$ (4) $\frac{-\pi i}{4}$
21.	The non-empty set of real numbers which is bounded below has (1) supremum (2) infimum (3) upper bound (4) none of these
22.	The sequence $\{f_n\}$ where $f_n(x) = x^n$ is ________ convergent on [0, k], k < 1 (1) uniformly (2) pointwise (3) nowhere (4) none of these
23.	Every bounded sequence has at least one limit point. This represents (1) Archimedean Property (2) Heine-Borel theorem (3) Bolzano-Weierstress theorem (4) Denseness Property
24.	Which of the following is convergent ? (1) $\sum_{n=1}^{\infty} n^2 2^{-n}$ (2) $\sum_{n=1}^{\infty} n^{-2} 2^n$ (3) $\sum_{n=2}^{\infty} \frac{1}{n \log n}$ (4) $\sum_{n=1}^{\infty} \frac{1}{n \log(1+1/n)}$
25.	If a function f defined on [0, 1] as $f(x) = \begin{cases} i, \text{if } x \neq 1/2 \\ 0, \text{if } x = 1/2 \end{cases}$, then (1) f is not bounded (2) f is R-integrable (3) f is not R-integrable since f is not bounded (4) f is not R-integrable since lower and upper limits are unequal

Question No.	Questions
26.	Let $f : R \rightarrow R$ be a monotone function. Then (1) f has no discontinuities (2) f has only finitely many discontinuities. (3) f can have at most countably many discontinuities (4) f can have uncountably many discontinuities
27.	The length of an interval I is (1) Outer measure of an interval I (2) Less than outer measure of an interval I (3) Greater than outer measure of an interval I (4) Twice the outer measure of an interval I
28.	A set E is said to be Lebesgue measurable if for each set A (1) $m^*(A) = m^*(A \cap E) - m^*(A \cap E^c)$ (2) $m^*(A) = m^*(A \cap E^c) - m^*(A \cap E)$ (3) $m^*(A) = m^*(A \cup E) + m^*(A \cap E^c)$ (4) $m^*(A) = m^*(A \cup E) - m^*(A \cap E^c)$
29.	A non-negative measurable function f is integrable over the measurable set E if (1) $\int_E f = \infty$ (2) $\int_E f > \infty$ (3) $\int_E f < \infty$ (4) None of these
30.	If f is of bounded variation on [a, b] and $c \in (a, b)$. Then (1) f is of bounded variation on [a, c] and on [c, b] (2) f is not of bounded variation on [a, c] and on [c, b] (3) f is constant on [a, c] and on [c, b] (4) None of these

Question No.	Questions
31.	A box contains N tickets which are numbered 1, 2, ..., N. Then value of N is however, unknown. A simple random sample of n tickets is drawn without replacement from the box. Let $X_1, X_2, ..., X_n$ be numbers on the tickets obtained in the $1^{st}, 2^{nd}, ..., n^{th}$ draws respectively. Here $\overline{X}=\frac{1}{N}(X_1+X_2+...+X_n)$. Which of the following is an unbiased estimator of N ? (1) $2\overline{X}-1$ (2) $2\overline{X}+1$ (3) $2\overline{X}+\frac{1}{2}$ (4) $2\overline{X}-\frac{1}{2}$
32.	Let $X_1 \sim N(0, 1)$ and let $X_2 = \begin{cases} -X_1, & -2 \le X_1 \le 2 \\ X_1, & \text{otherwise.} \end{cases}$ Then identify the correct statement. (1) corr $(X_1, X_2) = 1$ (2) X_2 does not have N (0, 1) distribution. (3) (X_1, X_2) has a bivariate normal distribution. (4) (X_1, X_2) does not have a bivariate normal distribution.
33.	Let $X_1, X_2, ..., X_n$ be a random sample of size n from a p-variate Normal distribution with mean μ and positive definite covariance matrix Σ. Choose the correct statement (1) $(X_1-\mu)' \Sigma^{-1} (X_1-\mu)$ has chi-suare distribution with 1 d.f. (2) $\overline{X}\,\overline{X}'$ has Wishart distribution with p d.f. (3) $\sum_{i=1}^{n}(X_i-\mu)(X_i-\mu)'$ has Wishart distribution with n d.f. (4) X_1+X_2 and X_1-X_2 are independently distributed.

Question No.	Questions
34.	In which of the following distributions, mean ≥ variance (1) Poisson distribution (2) Negative binomial distribution (3) Normal distribution (4) Binomial distribution
35.	Let X_1, X_2, ... be i.i.d. standard normal random variables and let $T_n = \frac{X_1^2 + ... + X_n^2}{n}$. Then (1) The limiting distribution of $T_n - 1$ is χ^2 with 1 degree of freedom. (2) The limiting distribution of $\frac{T_n - 1}{\sqrt{n}}$ is normal with mean 0 and variance 2. (3) The limiting distribution of $\sqrt{n}$ $(T_n - 1)$ is χ^2 with 1 degree of freedom. (4) The limiting distribution of $\sqrt{n}$ $(T_n - 1)$ is normal with mean 0 and variance 2.
36.	Suppose the cumulative distribution function of failure time T of a component is $1 - \exp(-ct^{\alpha})$, $t > 0$, $\alpha > 1$, $c > 0$. Then the hazard rate of λ (t) is (1) constant. (2) non-constant monotonically increasing in t. (3) non-constant monotonically decreasing in t. (4) not a monotone function in t.

Question No.	Questions
37.	Consider the following linear programming problem Maximize $z = 3x_1 + 2x_2$ subject to $x_1 + x_2 \geq 1$; $x_1 + x_2 \leq 5$; $2x_1 + 3x_2 \leq 6$; $-2x_1 + 3x_2 \leq 6$ The problem has (1) an unbounded solution (2) exactly one optimal solution (3) more than one optimal solution (4) no feasible solutions
38.	Let $\{X_n : n \geq 0\}$ be a Markov chain on a finite state space S with stationary transition probability matrix. Suppose that the chain is not irreducible. Then the Markov chain : (1) admits infinitely many stationary distributions (2) admits a unique stationary distribution (3) may not admit any stationary distribution (4) cannot admit exactly two stationary distributions
39.	Men arrive in a queue according to a Poisson process with rate λ_1 and women arrive in the same queue according to another Poisson process with rate λ_2. The arrivals of mean and women are independent. The probability that the first arrival in the queue is a man is : (1) $\frac{\lambda_1}{\lambda_1 + \lambda_2}$ (2) $\frac{\lambda_2}{\lambda_1 + \lambda_2}$ (3) $\frac{\lambda_1}{\lambda_2}$ (4) $\frac{\lambda_2}{\lambda_1}$

Question No.	Questions
40.	Let X (t) be the number of customers in an M/M/1 queuing system with arrival rate $\lambda > 0$ and service rate $\mu > 0$. The process X (t) is a (1) Poisson process with rate $\lambda - \mu$. (2) pure birth process with birth rate $\lambda - \mu$. (3) birth and death process with birth rate λ and death rate μ. (4) birth and death process with birth rate $1/\lambda$ and death rate $1/\mu$.
41.	The rate of convergence is faster for (1) Regula-Falsi method (2) Bisection method (3) Newton-Raphson method (4) Cannot say
42.	As soon as a new value of a variable is found by iteration, it is used immediately in the following equations, this method is called (1) Gauss-Jordan method (2) Gauss-Seidal method (3) Jacobi's method (4) Relaxation method
43.	The value of function f (x) at 4 discrete points are givne below : x: 0, 1, 2, 5 f (x): 2, 3, 12, 147 Using Lagrange's formula, the value of f (3) is (1) 30 (2) 35 (3) 25 (4) 20

Table for Question 43:

x	0	1	2	5
f (x)	2	3	12	147

Question No.	Questions
44.	The value of function f (x) at 5 discrete points are given below : x: 0, 0.1, 0.2, 0.3, 0.4 f (x): 0, 10, 40, 90, 160 Using Trapezoidal rule with step size of 0.1, the value of $\int_0^{0.4} f(x)\,dx$ is (1) 10.8 (2) 13.4 (3) 18.7 (4) 22.0
45.	If $y' = x + y$, $y(0) = 1$, $y_1(x) = 1 + x + \frac{x^2}{2}$, then by Picard's method, the value of $y_2(x)$ is : (1) $1 + x + x^2 + \frac{x^3}{6}$ (2) $1 - x + x^2 + \frac{x^3}{6}$ (3) $1 + x - x^2 + \frac{x^3}{6}$ (4) $1 + x + x^2 - \frac{x^3}{6}$
46.	$I = \int_{x_1}^{x_2} F(y, y')\,dx$ whose ends are fixed is stationary if y satisfies the equation (1) $\frac{\partial F}{\partial y'} = \text{constant}$ (2) $F - y' \frac{\partial F}{\partial y'} = \text{constant}$ (3) $F - y \frac{\partial F}{\partial y'} = \text{constant}$ (4) $F' - y \frac{\partial F}{\partial y'} = \text{constant}$

Question No.	Questions
47.	If $J[y] = \int_1^2 (y'^2 + 2yy' + y^2)\,dx$, y (1) = 1 and y (2) is arbitrary, then the extremal is (1) e^{x-1} (2) e^{x+1} (3) e^{1-x} (4) e^{-x-1}
48.	The extremal of $\int_1^2 \frac{\dot{x}^2}{t^3}\,dt$; x (1) = 3, x (2) = 18 $\left(\text{where } \dot{x} \equiv \frac{dx}{dt}\right)$ using Lagrange's equation is given by which of the following ? (1) $x = t^4 + 2$ (2) $x = \frac{15}{7}t^3 + \frac{6}{7}$ (3) $x = 5t^2 - 2$ (4) $x = 5t^3 + 3$
49.	The kernel sin (x + t) is (1) separable kernel (2) difference kernel (3) adjoint kernel (4) none of these
50.	The solution to the integral equation $\phi(x) = x + \int_0^x \sin(x-\xi)\,\phi(\xi)\,d\xi$ is given by (1) $x^2 + \frac{x^3}{3}$ (2) $x - \frac{x^3}{3!}$ (3) $x + \frac{x^3}{3!}$ (4) $x^2 - \frac{x^3}{3!}$

Question No.	Questions
51.	The statement, if f is entire and bounded for all $z \in C$, then f is constant, refers to : (1) Morera's theorem (2) Maximum modulus theorem (3) Liouville's theorem (4) Hurwitz theorem
52.	Let f(z) be analytic in a closed, connected domain, D then which of the following is not true (1) The extreme values of the modulus of the function must occur on the boundary (2) If $\|f(z)\|$ has an interior extrema, then the function is a constant (3) The extreme values of the modulus of the function may occur on interior point and f(z) may not be constant (4) If f(z) is non constant then $\|f(z)\|$ does not attains an extrema inside the boundary
53.	The function $f: C \to C$ defined by $f(z) = e^z + e^{-z}$ has (1) finitely many zeros (2) no zeros (3) only real zeros (4) has infinitely may zeros
54.	Consider the following complex function $f(z) = \frac{9}{(z-1)(z+2)^2}$. Which of the following is one of the residues of the above function ? (1) –1 (2) 9/16 (3) 2 (4) 9
55.	The bilinear transformation that maps the points $z = \infty, i, 0$ into the points $w = 0, i, \infty$ is (1) $w = -z$ (2) $w = z$ (3) $w = \frac{-1}{z}$ (4) $w = \frac{1}{z}$

Question No.	Questions
56.	The fundamental theorem of arithmetic states that (1) The factoring of any integer $n > 1$ into primes is not unique apart from the order of prime factors (2) The factoring of any integer $n > 1$ into primes is unique apart from the order of prime factors (3) There are infinitely many primes (4) The number of prime numbers is finite
57.	The last two digits of 7^{81} are (1) 07 (2) 17 (3) 37 (4) 47
58.	The congruence $35x \equiv 14 \pmod{21}$ has (1) 7 solutions (2) 6 solutions (3) 9 solutions (4) No solution
59.	If n is a positive integer such that the sum of all positive integers a satisfying $1 \leq a \leq n$ and GCD $(a, n) = 1$ is equal to 240n, then the number of summands, namely, $\phi(n)$, is (1) 120 (2) 124 (3) 240 (4) 480
60.	If gcd $(m, n) = 1$ where $m > 2$ and $n > 2$, then the integer mn has (1) no primitive roots (2) unique primitive root (3) infinite primitive roots (4) finite primitive roots
61.	The homogeneous integral equation $\phi(x) - \lambda \int_0^1 (3x-2)\, t\, \phi(t)\, dt = 0$, has (1) One characteristic number (2) Three characteristic numbers (3) Two characteristic numbers (4) No characteristic number

Question No.	Questions
62.	Let S be a mechanical system with Lagrangian L $(q_j, \dot{q}_j, t)$, j = 1, 2, ..., n and generalized coordinates. Then the Lagrange equations of motion for S (1) constitute a set of n first order ODEs. (2) can be transformed to the Hamilton form using Legendre transform. (3) are equivalent to a set on n first order ODEs when expressed in terms of Hamiltonian functions. (4) is a set of 2n second order ODEs.
63.	Lagrange's equations for a Holonomic dynamical system specified by n-generalized coordinates q_j (j = 1, 2, 3 ... n) having T as the K.E. of system at time t and Q_j the generalized forces are (1) $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right)+\frac{\partial T}{\partial q_j}=Q_j$ (2) $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right)-\frac{\partial T}{\partial q_j}=Q_j$ (3) $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right)-\frac{\partial T}{\partial q_j}=\dot{Q}_j$ (4) $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right)+\frac{\partial T}{\partial q_j}=\dot{Q}_j$
64.	Let q_i and $\dot{q}_i$ respectively are the generalized coordinates and velocity of a mechanical system and p_i are its generalized momenta. If H is the Hamiltonian of the system, then Hamilton's equations of motion are (1) $\dot{q}_i=\frac{\partial H}{\partial p_i}$, $\dot{p}_i=\frac{\partial H}{\partial q_i}$ (2) $\dot{q}_i=\frac{\partial H}{\partial p_i}$, $\dot{p}_i=-\frac{\partial H}{\partial q_i}$ (3) $\dot{q}_i=-\frac{\partial H}{\partial p_i}$, $\dot{p}_i=\frac{\partial H}{\partial q_i}$ (4) $\dot{q}_i=-\frac{\partial H}{\partial p_i}$, $\dot{p}_i=-\frac{\partial H}{\partial q_i}$

Question No.	Questions
65.	Hamiltonian H is defined as (1) $H = \sum p_i \dot{q}_i - L$ (2) $H = \sum \dot{p}_i q_i - L$ (3) $H = \sum \dot{p}_i q_i + L$ (4) $H = \sum \dot{p}_i \dot{q}_i - L$
66.	What is the probability to get two aces in succession (with replacement) from a deck of 52 cards ? (1) 1/52 (2) 1/169 (3) 2/159 (4) 2/169
67.	There are two boxes. Box 1 contains 2 red balls and 4 green balls. Box 2 contains 4 red balls and 2 green balls. A box is selected at random and a ball is chosen randomly from the selected box. If the ball turns out to be red, what is the probability that Box 1 had been selected ? (1) 1/2 (2) 1/6 (3) 2/3 (4) 1/3
68.	Suppose you have a coin with probability $3/4$ of getting a Head. You toss the coin twice independently. Let $\Omega = \{(H, H), (H, T), (T, H), (T, T)\}$ be the sample space. Then it is possible to have an event $E \subseteq \Omega$ such that (1) $P(E) = 1/3$ (2) $P(E) = 1/9$ (3) $P(E) = 1/4$ (4) $P(E) = 7/8$
69.	A random variable X has a probability distribution as follows : r: 0, 1, 2, 3 P (X = r): 2k, 3k, 13k, 2k Then the probability that P (X < 2) is equal to (1) 0.90 (2) 0.25 (3) 0.65 (4) 0.15

Table for Question 69:

r	0	1	2	3
P (X = r)	2k	3k	13k	2k

Question No.	Questions
70.	Find the value of λ such that the function f (x) is a valid probability density function where $f(x) = \begin{cases} \lambda (x-1)(2-x) & \text{for } 1 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$ (1) 1 (2) 5 (3) 6 (4) 7
71.	A continuous random variable X has a probability density function $f(x) = e^{-x}, 0 < x < \infty$. Then P {X > 1} is (1) 0.368 (2) 0.5 (3) 0.632 (4) 1.0
72.	Let X and Y be two random variables having the joint probability density function $f(x, y) = \begin{cases} 2 & \text{if } 0 < x < y < 1 \\ 0 & \text{otherwise.} \end{cases}$ Then the conditional probability $P\left(X \le \frac{2}{3} \middle| Y = \frac{3}{4}\right)$ is equal to (1) 5/9 (2) 2/3 (3) 7/9 (4) 8/9
73.	The variance of a random variable X is given by (1) $E[X - E(X)]^2$ (2) $[E(X)]^2 - E(X)^2$ (3) $E(X)^2 + (E(X))^2$ (4) None of these
74.	Standard Normal Variate has (1) Mean = 0 and variance = 1 (2) Mean = 1 and variance = 0 (3) Mean = 1 and variance = 1 (4) None of these

Question No.	Questions
75.	When $n \to \infty$, the Binomial distribution can be approximated as (1) Bernoulli distribution (2) Uniform distribution (3) Poisson distribution (4) None of these
76.	The variance of Poisson distribution is given by (1) $\sigma^2 = \lambda$ (2) $\sigma^2 = \frac{1}{\lambda}$ (3) $\sigma^2 = \frac{1}{\lambda^2}$ (4) None of these
77.	The first moment about origin is known as (1) Mean (2) Variance (3) Standard deviation (4) None of these
78.	In a hypothesis-testing problem, which of the following is not required in order to compute the p-value ? (1) Value of the test statistic (2) Distribution of the test statistic under the null hypothesis (3) The level of significance (4) Whether the test is one-sided or two-sided
79.	In testing $H : \mu = 100$ against $A : \mu \neq 100$ at the 10% level of significance, H is rejected if (1) 100 is contained in the 90% confidence interval (2) The value of the test statistic is in the acceptance region (3) The p-value is less than 0.10 (4) The p-value is greater than 0.10

Question No.	Questions
80.	In the context of testing of statistical hypothesis, which one of the following statements is true ? (1) When testing a simple hypothesis H_0 against an alternative simple hypothesis H_1, the likelihood ratio principle leads to the most powerful test. (2) When testing a simple hypothesis H_0 against an alternative simple hypothesis H_1, P [rejecting $H_0 \mid H_0$ is true] + P [accepting $H_0 \mid H_1$ is true] = 1. (3) For testing a simple hypothesis H_0 against an alternative simple hypothesis H_1, randomized test is used to achieve the desired level of the power of the test. (4) UMP test for testing a simple hypothesis H_0 against an alternative simple hypothesis H_1, always exist.
81.	Let X and Y be metric spaces, and $f : X \to Y$ a function then which of the following is true (1) f is continuous ; (2) for every open set U in Y, f^{-1} (U) is open in X (3) for every closed set C in Y, f^{-1} (C) is closed in X (4) All the above
82.	The metric space (R, d), where d is a usual metric, is (1) compact (2) disconnected (3) connected but not compact (4) compact and connected
83.	Let (X, d) be a metric space, then for all x, y, z $\in$X (1) $d(x, y) \leq d(x, z) + d(z, y)$ (2) $d(x, y) \geq d(x, z) + d(z, y)$ (3) $d(x, y) \leq 0$ (4) None of these

Question No.	Questions
84.	A normed linear space X is complete iff (1) Every convergent series in X is absolutely convergent (2) Every convergent series in X is convergent (3) Every convergent series in X is uniformly convergent (4) Every absolutely convergent series in X is convergent
85.	If W_1, W_2 are two subspaces of a finite dimension vector space V(F), then (1) $\dim (W_1 + W_2) = \dim (W_1 \cup W_2)$ (2) $\dim (W_1 + W_2) = \dim W_1 + \dim W_2$ (3) $\dim (W_1 + W_2) = \dim W_1 + \dim W_2 - \dim (W_1 \cap W_2)$ (4) $\dim (W_1 + W_2) = \dim W_1 + \dim W_2 + \dim (W_1 \cap W_2)$
86.	The co-ordinates of vector (1, 1, 1) relative to basis (1, 1, 2), (2, 2, 1), (1, 2, 2) is (1) (1/3, 0, 1/3) (2) (1/3, 1/3, 0) (3) (0, 1/3, 1/3) (4) None of these
87.	Let $T : R^n \to R^n$ be a linear transformation. Which of the following statements implies that T is bijective ? (1) Nullity (T) = n (2) Rank (T) = Nullity (T) = n (3) Rank (T) + Nullity (T) = n (4) Rank (T) – Nullity (T) = n
88.	Let A, B be $n \times n$ real matrices. Which of the following statements is correct ? (1) rank (A + B) = rank(A) + rank(B) (2) rank (A + B) $\leq$ rank(A) + rank(B) (3) rank (A + B) = min {rank(A), rank(B)} (4) rank (A + B) = max {rank (A), rank(B)}

Question No.	Questions
89.	Let A and B be real invertible matrices such that AB = – BA. Then (1) Trace (A) = 1, Trace (B) = 0 (2) Trace (A) = Trace (B) = 1 (3) Trace (A) = 0, Trace (B) = 1 (4) Trace (A) = Trace (B) = 0
90.	Consider the matrix $A(x) = \begin{bmatrix} 1+x^2 & 7 & 11 \\ 3x & 2x & 4 \\ 8x & 17 & 13 \end{bmatrix}$; $x \in R$. Then (1) A (x) has eigenvalue 0 for some $x \in R$ (2) 0 is not an eigenvalue of A(x) for any $x \in R$ (3) A (x) has eigenvalue 0 for all $x \in R$ (4) A (x) is invertible for every $x \in R$
91.	Consider the initial value problem (IVP) $\frac{dy}{dx} = y^2$, $y(0) = 1$, $(x, y) \in R \times R$. Then there exists a unique solution of the IVP on (1) $(-\infty, \infty)$ (2) $(-\infty, 1)$ (3) $(-2, 2)$ (4) $(-1, \infty)$
92.	The solution to the initial value problem $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 5y = 3e^{-t} \sin t$, $y(0) = 0$ and $y'(0) = 3$, is (1) $y(t) = e^{t} (\sin t + \sin 2t)$ (2) $y(t) = e^{-t} (\sin t + \sin 2t)$ (3) $y(t) = 3 e^{t} \sin t$ (4) $y(t) = 3 e^{-t} \sin t$

Question No.	Questions
93.	Consider the differential equation $(x-1)y'' + xy' + \frac{1}{x}y = 0$. Then (1) $x = 1$ is the only singular point (2) $x = 0$ is the only singular point (3) both $x = 0$ and $x = 1$ are singular points (4) neither $x = 0$ nor $x = 1$ are singular points
94.	Let f and g be real linearly independent solutions of $\frac{d}{dx}\left[\frac{dy}{dx}P(x)\right] + Q(x)y = 0$ on the interval $a \le x \le b$. Then (1) between any two consecutive zeros of f, there is precisely one zero of g. (2) between any two consecutive zeros of f, there is no zero of g. (3) between any two consecutive zeros of f, there is infinite zeros of g. (4) None of these
95.	The eigen values of a Sturm-Liouville BVP are (1) Always positive (2) Always negative (3) Always real (4) Always in the pair of complex conjugate

Question No.	Questions
96.	The Charpit's equations for the PDE $up^2 + q^2 + x + y = 0$, $p = \frac{\partial u}{\partial x}$, $q = \frac{\partial u}{\partial y}$ are given by (1) $\frac{dx}{-1-p^3} = \frac{dy}{-1-qp^2} = \frac{du}{2p^2u+2q^2} = \frac{dp}{2pu} = \frac{dq}{2q}$ (2) $\frac{dx}{2pu} = \frac{dy}{2q} = \frac{du}{2p^2u+2q^2} = \frac{dp}{-1-p^3} = \frac{dq}{-1-qp^2}$ (3) $\frac{dx}{up^2} = \frac{dy}{q^2} = \frac{du}{0} = \frac{dp}{x} = \frac{dq}{y}$ (4) $\frac{dx}{2q} = \frac{dy}{2pu} = \frac{du}{x+y} = \frac{dp}{p^2} = \frac{dq}{qp^2}$
97.	The partial differential equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + u$ can be transformed to $\frac{\partial v}{\partial t} = \frac{\partial^2 v}{\partial x^2}$ for (1) $v = e^{-t} u$. (2) $v = e^{t} u$. (3) $v = tu$. (4) $v = -tu$.
98.	The PDE $x\frac{\partial^2 u}{\partial x^2} + y\frac{\partial^2 u}{\partial y^2} = 0$ is (1) elliptic for $x < 0, y > 0$ (2) hyperbolic for $x > 0, y < 0$ (3) elliptic for $x > 0, y < 0$ (4) hyperbolic for $x > 0, y > 0$

Question No.	Questions
99.	Solution of $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2} = 0$ is given by (1) $z = f_1(y + 3x) + f_2(y + 2x)$ (2) $z = f_1(y + 3x) + f_2(y - 2x)$ (3) $z = f_1(y - 2x) + f_2(y + 2x)$ (4) None of these
100.	Given Wave equation $\frac{\partial z}{\partial y} = c^2 \frac{\partial^2 z}{\partial x^2}$. By separation of variable, let $z(x, y) = X(x)\,Y(y)$ be a solution. Substituting it in $\frac{\partial z}{\partial y} = c^2 \frac{\partial^2 z}{\partial x^2}$ one have $\frac{d^2X}{dx^2} + kX = 0$ and $\frac{dY}{dy} - kc^2 Y = 0$. If $k = -p^2$, p is real, then the solution is (c's are constants) (1) $z(x, y) = (c_1 \cos px)\, c_2\, e^{-c^2 p^2 y}$ (2) $z(x, y) = (c_1 \cos px + c_2 \sin px)\, c_1\, e^{-c^2 p^2 y}$ (3) $z(x, y) = e^{-c^2 p^2 y}$ (4) $z(x, y) = (c_2 \cos px)\, c_1\, e^{-c^2 p^2 y}$

Question No.	Questions
1.	The homogeneous integral equation $\phi(x) - \lambda \int_0^1 (3x-2)\, t\, \phi(t)\, dt = 0$, has (1) One characteristic number (2) Three characteristic numbers (3) Two characteristic numbers (4) No characteristic number
2.	Let S be a mechanical system with Lagrangian L $(q_j, \dot{q}_j, t)$, j = 1, 2, ..., n and generalized coordinates. Then the Lagrange equations of motion for S (1) constitute a set of n first order ODEs. (2) can be transformed to the Hamilton form using Legendre transform. (3) are equivalent to a set on n first order ODEs when expressed in terms of Hamiltonian functions. (4) is a set of 2n second order ODEs.
3.	Lagrange's equations for a Holonomic dynamical system specified by n-generalized coordinates q_j (j = 1, 2, 3 ... n) having T as the K.E. of system at time t and Q_j the generalized forces are (1) $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) + \frac{\partial T}{\partial q_j} = Q_j$ (2) $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} = Q_j$ (3) $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} = \dot{Q}_j$ (4) $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) + \frac{\partial T}{\partial q_j} = \dot{Q}_j$

Question No.	Questions
4.	Let q_i and $\dot{q}_i$ respectively are the generalized coordinates and velocity of a mechanical system and p_i are its generalized momenta. If H is the Hamiltonian of the system, then Hamilton's equations of motion are (1) $\dot{q}_i = \frac{\partial H}{\partial p_i}, \ \dot{p}_i = \frac{\partial H}{\partial q_i}$ (2) $\dot{q}_i = \frac{\partial H}{\partial p_i}, \ \dot{p}_i = -\frac{\partial H}{\partial q_i}$ (3) $\dot{q}_i = -\frac{\partial H}{\partial p_i}, \ \dot{p}_i = \frac{\partial H}{\partial q_i}$ (4) $\dot{q}_i = -\frac{\partial H}{\partial p_i}, \ \dot{p}_i = -\frac{\partial H}{\partial q_i}$
5.	Hamiltonian H is defined as (1) $H = \sum p_i \dot{q}_i - L$ (2) $H = \sum \dot{p}_i q_i - L$ (3) $H = \sum \dot{p}_i q_i + L$ (4) $H = \sum \dot{p}_i \dot{q}_i - L$
6.	What is the probability to get two aces in succession (with replacement) from a deck of 52 cards ? (1) 1/52 (2) 1/169 (3) 2/159 (4) 2/169
7.	There are two boxes. Box 1 contains 2 red balls and 4 green balls. Box 2 contains 4 red balls and 2 green balls. A box is selected at random and a ball is chosen randomly from the selected box. If the ball turns out to be red, what is the probability that Box 1 had been selected ? (1) 1/2 (2) 1/6 (3) 2/3 (4) 1/3

Question No.	Questions
8.	Suppose you have a coin with probability $\frac{3}{4}$ of getting a Head. You toss the coin twice independently. Let $\Omega = \{(H, H), (H, T), (T, H), (T, T)\}$ be the sample space. Then it is possible to have an event $E \subseteq \Omega$ such that (1) $P(E) = 1/3$ (2) $P(E) = 1/9$ (3) $P(E) = 1/4$ (4) $P(E) = 7/8$
9.	A random variable X has a probability distribution as follows : r: 0, 1, 2, 3 P (X = r): 2k, 3k, 13k, 2k Then the probability that P (X < 2) is equal to (1) 0.90 (2) 0.25 (3) 0.65 (4) 0.15
10.	Find the value of λ such that the function f (x) is a valid probability density function where $f(x) = \begin{cases} \lambda (x-1)(2-x) & \text{for } 1 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$ (1) 1 (2) 5 (3) 6 (4) 7
11.	Consider the initial value problem (IVP) $\frac{dy}{dx} = y^2,\ y(0) = 1,\ (x, y) \in R \times R.$ Then there exists a unique solution of the IVP on (1) $(-\infty, \infty)$ (2) $(-\infty, 1)$ (3) $(-2, 2)$ (4) $(-1, \infty)$

Table for Question 9:

r	0	1	2	3
P (X = r)	2k	3k	13k	2k

Question No.	Questions
12.	The solution to the initial value problem $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 5y = 3e^{-t} \sin t$, $y(0) = 0$ and $y'(0) = 3$, is (1) $y(t) = e^{t}(\sin t + \sin 2t)$ (2) $y(t) = e^{-t}(\sin t + \sin 2t)$ (3) $y(t) = 3e^{t} \sin t$ (4) $y(t) = 3e^{-t} \sin t$
13.	Consider the differential equation $(x-1)y'' + xy' + \frac{1}{x}y = 0$. Then (1) $x = 1$ is the only singular point (2) $x = 0$ is the only singular point (3) both $x = 0$ and $x = 1$ are singular points (4) neither $x = 0$ nor $x = 1$ are singular points
14.	Let f and g be real linearly independent solutions of $\frac{d}{dx}\left[\frac{dy}{dx}P(x)\right] + Q(x)y = 0$ on the interval $a \le x \le b$. Then (1) between any two consecutive zeros of f, there is precisely one zero of g. (2) between any two consecutive zeros of f, there is no zero of g. (3) between any two consecutive zeros of f, there is infinite zeros of g. (4) None of these
15.	The eigen values of a Sturm-Liouville BVP are (1) Always positive (2) Always negative (3) Always real (4) Always in the pair of complex conjugate

Question No.	Questions
16.	The Charpit's equations for the PDE $up^2 + q^2 + x + y = 0$, $p = \frac{\partial u}{\partial x}$, $q = \frac{\partial u}{\partial y}$ are given by (1) $\frac{dx}{-1-p^3} = \frac{dy}{-1-qp^2} = \frac{du}{2p^2u+2q^2} = \frac{dp}{2pu} = \frac{dq}{2q}$ (2) $\frac{dx}{2pu} = \frac{dy}{2q} = \frac{du}{2p^2u+2q^2} = \frac{dp}{-1-p^3} = \frac{dq}{-1-qp^2}$ (3) $\frac{dx}{up^2} = \frac{dy}{q^2} = \frac{du}{0} = \frac{dp}{x} = \frac{dq}{y}$ (4) $\frac{dx}{2q} = \frac{dy}{2pu} = \frac{du}{x+y} = \frac{dp}{p^2} = \frac{dq}{qp^2}$
17.	The partial differential equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + u$ can be transformed to $\frac{\partial v}{\partial t} = \frac{\partial^2 v}{\partial x^2}$ for (1) $v = e^{-t}u$. (2) $v = e^{t}u$. (3) $v = tu$. (4) $v = -tu$.
18.	The PDE $x\frac{\partial^2 u}{\partial x^2} + y\frac{\partial^2 u}{\partial y^2} = 0$ is (1) elliptic for $x < 0, y > 0$ (2) hyperbolic for $x > 0, y < 0$ (3) elliptic for $x > 0, y < 0$ (4) hyperbolic for $x > 0, y > 0$

Question No.	Questions
19.	Solution of $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2} = 0$ is given by (1) $z = f_1(y + 3x) + f_2(y + 2x)$ (2) $z = f_1(y + 3x) + f_2(y - 2x)$ (3) $z = f_1(y - 2x) + f_2(y + 2x)$ (4) None of these
20.	Given Wave equation $\frac{\partial z}{\partial y} = c^2 \frac{\partial^2 z}{\partial x^2}$. By separation of variable, let $z(x, y) = X(x)\,Y(y)$ be a solution. Substituting it in $\frac{\partial z}{\partial y} = c^2 \frac{\partial^2 z}{\partial x^2}$ one have $\frac{d^2X}{dx^2} + kX = 0$ and $\frac{dY}{dy} - kc^2 Y = 0$. If $k = -p^2$, p is real, then the solution is (c's are constants) (1) $z(x, y) = (c_1 \cos px)\, c_2\, e^{-c^2 p^2 y}$ (2) $z(x, y) = (c_1 \cos px + c_2 \sin px)\, c_1\, e^{-c^2 p^2 y}$ (3) $z(x, y) = e^{-c^2 p^2 y}$ (4) $z(x, y) = (c_2 \cos px)\, c_1\, e^{-c^2 p^2 y}$
21.	The statement, if *f* is entire and bounded for all $z \in C$, then *f* is constant, refers to : (1) Morera's theorem (2) Maximum modulus theorem (3) Liouville's theorem (4) Hurwitz theorem

Question No.	Questions
22.	Let f(z) be analytic in a closed, connected domain, D then which of the following is not true (1) The extreme values of the modulus of the function must occur on the boundary (2) If $\lvert f(z) \rvert$ has an interior extrema, then the function is a constant (3) The extreme values of the modulus of the function may occur on interior point and f(z) may not be constant (4) If f(z) is non constant then $\lvert f(z) \rvert$ does not attains an extrema inside the boundary
23.	The function $f : C \to C$ defined by $f(z) = e^{z} + e^{-z}$ has (1) finitely many zeros (2) no zeros (3) only real zeros (4) has infinitely may zeros
24.	Consider the following complex function $f(z) = \frac{9}{(z-1)(z+2)^2}$. Which of the following is one of the residues of the above function ? (1) –1 (2) 9/16 (3) 2 (4) 9
25.	The bilinear transformation that maps the points z = ∞, i, 0 into the points w = 0, i, ∞ is (1) $w = -z$ (2) $w = z$ (3) $w = \frac{-1}{z}$ (4) $w = \frac{1}{z}$

Question No.	Questions
26.	The fundamental theorem of arithmetic states that (1) The factoring of any integer $n > 1$ into primes is not unique apart from the order of prime factors (2) The factoring of any integer $n > 1$ into primes is unique apart from the order of prime factors (3) There are infinitely many primes (4) The number of prime numbers is finite
27.	The last two digits of 7^{81} are (1) 07 (2) 17 (3) 37 (4) 47
28.	The congruence $35x \equiv 14 \pmod{21}$ has (1) 7 solutions (2) 6 solutions (3) 9 solutions (4) No solution
29.	If n is a positive integer such that the sum of all positive integers a satisfying $1 \leq a \leq n$ and GCD $(a, n) = 1$ is equal to 240n, then the number of summands, namely, $\phi(n)$, is (1) 120 (2) 124 (3) 240 (4) 480
30.	If gcd $(m, n) = 1$ where $m > 2$ and $n > 2$, then the integer mn has (1) no primitive roots (2) unique primitive root (3) infinite primitive roots (4) finite primitive roots
31.	Let X and Y be metric spaces, and $f : X \rightarrow Y$ a function then which of the following is true (1) f is continuous ; (2) for every open set U in Y, $f^{-1}(U)$ is open in X (3) for every closed set C in Y, $f^{-1}(C)$ is closed in X (4) All the above

Question No.	Questions
32.	The metric space (R, d), where d is a usual metric, is (1) compact (2) disconnected (3) connected but not compact (4) compact and connected
33.	Let (X, d) be a metric space, then for all x, y, z $\in$X (1) $d(x, y) \leq d(x, z) + d(z, y)$ (2) $d(x, y) \geq d(x, z) + d(z, y)$ (3) $d(x, y) \leq 0$ (4) None of these
34.	A normed linear space X is complete iff (1) Every convergent series in X is absolutely convergent (2) Every convergent series in X is convergent (3) Every convergent series in X is uniformly convergent (4) Every absolutely convergent series in X is convergent
35.	If W_1, W_2 are two subspaces of a finite dimension vector space V(F), then (1) $\dim (W_1 + W_2) = \dim (W_1 \cup W_2)$ (2) $\dim (W_1 + W_2) = \dim W_1 + \dim W_2$ (3) $\dim (W_1 + W_2) = \dim W_1 + \dim W_2 - \dim (W_1 \cap W_2)$ (4) $\dim (W_1 + W_2) = \dim W_1 + \dim W_2 + \dim (W_1 \cap W_2)$
36.	The co-ordinates of vector (1, 1, 1) relative to basis (1, 1, 2), (2, 2, 1), (1, 2, 2) is (1) (1/3, 0, 1/3) (2) (1/3, 1/3, 0) (3) (0, 1/3, 1/3) (4) None of these
37.	Let $T : R^n \to R^n$ be a linear transformation. Which of the following statements implies that T is bijective ? (1) Nullity (T) = n (2) Rank (T) = Nullity (T) = n (3) Rank (T) + Nullity (T) = n (4) Rank (T) – Nullity (T) = n

Question No.	Questions
38.	Let A, B be n × n real matrices. Which of the following statements is correct ? (1) rank (A + B) = rank(A) + rank(B) (2) rank (A + B) $\leq$ rank(A) + rank(B) (3) rank (A + B) = min {rank(A), rank(B)} (4) rank (A + B) = max {rank (A), rank(B)}
39.	Let A and B be real invertible matrices such that AB = – BA. Then (1) Trace (A) = 1, Trace (B) = 0 (2) Trace (A) = Trace (B) = 1 (3) Trace (A) = 0, Trace (B) = 1 (4) Trace (A) = Trace (B) = 0
40.	Consider the matrix $A(x) = \begin{bmatrix} 1+x^2 & 7 & 11 \\ 3x & 2x & 4 \\ 8x & 17 & 13 \end{bmatrix}$; $x \in R$. Then (1) A (x) has eigenvalue 0 for some $x \in R$ (2) 0 is not an eigenvalue of A(x) for any $x \in R$ (3) A (x) has eigenvalue 0 for all $x \in R$ (4) A (x) is invertible for every $x \in R$
41.	A box contains N tickets which are numbered 1, 2, ..., N. Then value of N is however, unknown. A simple random sample of n tickets is drawn without replacement from the box. Let $X_1, X_2, ..., X_n$ be numbers on the tickets obtained in the 1st, 2nd, ..., nth draws respectively. Here $\overline{X} = \frac{1}{N}(X_1 + X_2 + ... + X_n)$. Which of the following is an unbiased estimator of N ? (1) $2\overline{X} - 1$ (2) $2\overline{X} + 1$ (3) $2\overline{X} + \frac{1}{2}$ (4) $2\overline{X} - \frac{1}{2}$

Question No.	Questions
42.	Let $X_1 \sim N(0, 1)$ and let $X_2 = \begin{cases} -X_1, & -2 \le X_1 \le 2 \\ X_1, & \text{otherwise.} \end{cases}$ Then identify the correct statement. (1) $\text{corr}(X_1, X_2) = 1$ (2) X_2 does not have N (0, 1) distribution. (3) (X_1, X_2) has a bivariate normal distribution. (4) (X_1, X_2) does not have a bivariate normal distribution.
43.	Let $X_1, X_2, ..., X_n$ be a random sample of size n from a p-variate Normal distribution with mean μ and positive definite covariance matrix Σ. Choose the correct statement (1) $(X_1 - \mu)' \sum^{-1} (X_1 - \mu)$ has chi-suare distribution with 1 d.f. (2) $\overline{X}\,\overline{X}'$ has Wishart distribution with p d.f. (3) $\sum_{i=1}^{n} (X_i - \mu)(X_i - \mu)'$ has Wishart distribution with n d.f. (4) $X_1 + X_2$ and $X_1 - X_2$ are independently distributed.
44.	In which of the following distributions, mean $\geq$ variance (1) Poisson distribution (2) Negative binomial distribution (3) Normal distribution (4) Binomial distribution

Question No.	Questions
45.	Let X_1, X_2, ... be i.i.d. standard normal random variables and let $T_n = \frac{X_1^2 + ... + X_n^2}{n}$. Then (1) The limiting distribution of $T_n - 1$ is χ^2 with 1 degree of freedom. (2) The limiting distribution of $\frac{T_n - 1}{\sqrt{n}}$ is normal with mean 0 and variance 2. (3) The limiting distribution of $\sqrt{n}\ (T_n - 1)$ is χ^2 with 1 degree of freedom. (4) The limiting distribution of $\sqrt{n}\ (T_n - 1)$ is normal with mean 0 and variance 2.
46.	Suppose the cumulative distribution function of failure time T of a component is $1 - \exp(-ct^{\alpha}),\ \ t > 0,\ \alpha > 1,\ c > 0.$ Then the hazard rate of λ (t) is (1) constant. (2) non-constant monotonically increasing in t. (3) non-constant monotonically decreasing in t. (4) not a monotone function in t.
47.	Consider the following linear programming problem Maximize. $z = 3x_1 + 2x_2$ subject to $x_1 + x_2 \geq 1$; $x_1 + x_2 \leq 5$; $2x_1 + 3x_2 \leq 6$; $-2x_1 + 3x_2 \leq 6$ The problem has (1) an unbounded solution (2) exactly one optimal solution (3) more than one optimal solution (4) no feasible solutions

Question No.	Questions
48.	Let $\{X_n : n \geq 0\}$ be a Markov chain on a finite state space S with stationary transition probability matrix. Suppose that the chain is not irreducible. Then the Markov chain : (1) admits infinitely many stationary distributions (2) admits a unique stationary distribution (3) may not admit any stationary distribution (4) cannot admit exactly two stationary distributions
49.	Men arrive in a queue according to a Poisson process with rate λ_1 and women arrive in the same queue according to another Poisson process with rate λ_2. The arrivals of mean and women are independent. The probability that the first arrival in the queue is a man is : (1) $\frac{\lambda_1}{\lambda_1+\lambda_2}$ (2) $\frac{\lambda_2}{\lambda_1+\lambda_2}$ (3) $\frac{\lambda_1}{\lambda_2}$ (4) $\frac{\lambda_2}{\lambda_1}$
50.	Let X (t) be the number of customers in an M/M/1 queuing system with arrival rate $\lambda > 0$ and service rate $\mu > 0$. The process X (t) is a (1) Poisson process with rate $\lambda - \mu$. (2) pure birth process with birth rate $\lambda - \mu$. (3) birth and death process with birth rate λ and death rate μ. (4) birth and death process with birth rate $1/\lambda$ and death rate $1/\mu$.
51.	The rate of convergence is faster for (1) Regula-Falsi method (2) Bisection method (3) Newton-Raphson method (4) Cannot say

Question No.	Questions
52.	As soon as a new value of a variable is found by iteration, it is used immediately in the following equations, this method is called (1) Gauss-Jordan method (2) Gauss-Seidal method (3) Jacobi's method (4) Relaxation method
53.	The value of function f (x) at 4 discrete points are givne below : <table><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>5</td></tr><tr><td>f (x)</td><td>2</td><td>3</td><td>12</td><td>147</td></tr></table>Using Lagrange's formula, the value of f (3) is (1) 30 (2) 35 (3) 25 (4) 20
54.	The value of function f (x) at 5 discrete points are given below : <table><tr><td>x</td><td>0</td><td>0.1</td><td>0.2</td><td>0.3</td><td>0.4</td></tr><tr><td>f (x)</td><td>0</td><td>10</td><td>40</td><td>90</td><td>160</td></tr></table>Using Trapezoidal rule with step size of 0.1, the value of $\int_0^{0.4} f(x)\,dx$ is (1) 10.8 (2) 13.4 (3) 18.7 (4) 22.0
55.	If $y' = x + y$, $y(0) = 1$, $y_1(x) = 1 + x + \frac{x^2}{2}$, then by Picard's method, the value of $y_2(x)$ is : (1) $1 + x + x^2 + \frac{x^3}{6}$ (2) $1 - x + x^2 + \frac{x^3}{6}$ (3) $1 + x - x^2 + \frac{x^3}{6}$ (4) $1 + x + x^2 - \frac{x^3}{6}$

Question No.	Questions
56.	$I = \int_{x_1}^{x_2} F(y, y')\,dx$ whose ends are fixed is stationary if y satisfies the equation (1) $\frac{\partial F}{\partial y'} = \text{constant}$ (2) $F - y' \frac{\partial F}{\partial y'} = \text{constant}$ (3) $F - y \frac{\partial F}{\partial y'} = \text{constant}$ (4) $F' - y \frac{\partial F}{\partial y'} = \text{constant}$
57.	If $J[y] = \int_1^2 (y'^2 + 2yy' + y^2)\,dx$, y (1) = 1 and y (2) is arbitrary, then the extremal is (1) e^{x-1} (2) e^{x+1} (3) e^{1-x} (4) e^{-x-1}
58.	The extremal of $\int_1^2 \frac{\dot{x}^2}{t^3}\,dt$; x (1) = 3, x (2) = 18 $\left(\text{where } \dot{x} \equiv \frac{dx}{dt}\right)$ using Lagrange's equation is given by which of the following ? (1) $x = t^4 + 2$ (2) $x = \frac{15}{7} t^3 + \frac{6}{7}$ (3) $x = 5t^2 - 2$ (4) $x = 5t^3 + 3$
59.	The kernel sin (x + t) is (1) separable kernel (2) difference kernel (3) adjoint kernel (4) none of these

Question No.	Questions
60.	The solution to the integral equation $\phi(x) = x + \int_0^x \sin(x-\xi)\phi(\xi)\,d\xi$ is given by (1) $x^2 + \frac{x^3}{3}$ (2) $x - \frac{x^3}{3!}$ (3) $x + \frac{x^3}{3!}$ (4) $x^2 - \frac{x^3}{3!}$
61.	A continuous random variable X has a probability density function $f(x) = e^{-x}, 0 < x < \infty$. Then P $\{X > 1\}$ is (1) 0.368 (2) 0.5 (3) 0.632 (4) 1.0
62.	Let X and Y be two random variables having the joint probability density function $f(x, y) = \begin{cases} 2 & \text{if } 0 < x < y < 1 \\ 0 & \text{otherwise.} \end{cases}$ Then the conditional probability $P\left(X \le \frac{2}{3} \middle\vert Y = \frac{3}{4}\right)$ is equal to (1) 5/9 (2) 2/3 (3) 7/9 (4) 8/9
63.	The variance of a random variable X is given by (1) $E[X - E(X)]^2$ (2) $[E(X)]^2 - E(X)^2$ (3) $E(X)^2 + (E(X))^2$ (4) None of these

Question No.	Questions
64.	Standard Normal Variate has (1) Mean = 0 and variance = 1 (2) Mean = 1 and variance = 0 (3) Mean = 1 and variance = 1 (4) None of these
65.	When n $\rightarrow \infty$, the Binomial distribution can be approximated as (1) Bernoulli distribution (2) Uniform distribution (3) Poisson distribution (4) None of these
66.	The variance of Poisson distribution is given by (1) $\sigma^2 = \lambda$ (2) $\sigma^2 = \frac{1}{\lambda}$ (3) $\sigma^2 = \frac{1}{\lambda^2}$ (4) None of these
67.	The first moment about origin is known as (1) Mean (2) Variance (3) Standard deviation (4) None of these
68.	In a hypothesis-testing problem, which of the following is not required in order to compute the p-value ? (1) Value of the test statistic (2) Distribution of the test statistic under the null hypothesis (3) The level of significance (4) Whether the test is one-sided or two-sided

Question No.	Questions
69.	In testing H : $\mu = 100$ against A : $\mu \neq 100$ at the 10% level of significance, H is rejected if (1) 100 is contained in the 90% confidence interval (2) The value of the test statistic is in the acceptance region (3) The p-value is less than 0.10 (4) The p-value is greater than 0.10
70.	In the context of testing of statistical hypothesis, which one of the following statements is true ? (1) When testing a simple hypothesis H_0 against an alternative simple hypothesis H_1, the likelihood ratio principle leads to the most powerful test. (2) When testing a simple hypothesis H_0 against an alternative simple hypothesis H_1, P [rejecting $H_0 \mid H_0$ is true] + P [accepting $H_0 \mid H_1$ is true] = 1. (3) For testing a simple hypothesis H_0 against an alternative simple hypothesis H_1, randomized test is used to achieve the desired level of the power of the test. (4) UMP test for testing a simple hypothesis H_0 against an alternative simple hypothesis H_1, always exist.
71.	If p is a prime, then any group G of order 2p has (1) a normal subgroup of order p (2) a normal subgroup of order 2p (3) a normal subgroup of order p^2 (4) None of these
72.	Let G be simple group of order 60. Then (1) G has six Sylow-5 subgroups (2) G has four Sylow-3 subgroups (3) G has a cyclic subgroup of order 6 (4) G has a unique element of order 2

Question No.	Questions
73.	Let R be a Euclidean domain such that R is not a field. Then the polynomial ring R[X] is always (1) a Euclidean domain (2) a principal ideal domain but not a Euclidean domain (3) a unique factorization domain but not a principal ideal domain (4) not a unique factorization domain
74.	Let $p(x) = 9x^5 + 10x^3 + 5x + 15$ and $q(x) = x^3 - x^2 - x - 2$ be two polynomials in Q[x]. Then over Q, (1) p(x) and q(x) are both irreducible (2) p(x) is reducible but q(x) is irreducible (3) p(x) is irreducible but q(x) is reducible (4) p(x) and q(x) are both reducible
75.	Find the degree of the field extension $Q\left(\sqrt{2}, \sqrt[4]{2}, \sqrt[8]{2}\right)$ over Q. (1) 4 (2) 8 (3) 14 (4) 32
76.	Let F be a finite field and let K/F be a field extension of degree 6. Then the Galois group of K/F is isomorphic to (1) the cyclic group of order 6 (2) the permutation group of {1, 2, 3} (3) the permutation group on {1, 2, 3, 4, 5, 6} (4) the permutation group on {1}
77.	Let X be a topological space and A be a subset of X, then X is separable if (1) A is countable and $\overline{A} = X$ (2) $\overline{A}$ is countable (3) A is uncountable (4) None of these

Question No.	Questions
78.	Which of the following spaces is not separable ? (1) R with the trivial topology (2) The Cantor set as a subspace of R (3) R with the discrete topology (4) None of these
79.	Which of the following is true ? (1) Let X be compact and $f : X \rightarrow R$ be locally bounded. Then f is not bounded. (2) Closed subspaces of compact spaces are compact (3) Closed subspaces of compact spaces may not be compact (4) Continuous images of compact spaces may not be compact
80.	Let X and Y be two topological spaces and let $f : X \rightarrow Y$ be a continuous function. Then (1) f (K) is connected if $K \subset X$ is connected (2) f^{-1} (K) is connected if $K \subset Y$ is connected (3) f^{-1} (K) is compact if $K \subset Y$ is compact (4) None of these
81.	The Linear transformation $T : R^3 \rightarrow R^3$ corresponding to the matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ is (1) $T(x_1, x_2, x_3) = (x_1, 2x_2, 3x_3)$ (2) $T(x_1, x_2, x_3) = (x_1 + x_3, 2x_1 + x_2, x_2 + x_3)$ (3) $T(x_1, x_2, x_3) = (x_1, x_2, x_3)$ (4) None of these

Question No.	Questions
82.	The matrix $\begin{bmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix}$ is (1) non-negative definite but not positive definite (2) positive definite (3) negative definite (4) neither negative definite nor positive definite
83.	Let $X = \begin{bmatrix} 2 & 0 & -3 \\ 3 & -1 & -3 \\ 0 & 0 & -1 \end{bmatrix}$. A matrix P such that $P^{-1}XP$ is a diagonal matrix, is (1) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ (2) $\begin{bmatrix} -1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ (3) $\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ (4) $\begin{bmatrix} -1 & -1 & 1 \\ 0 & -1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$
84.	The norm of x with respect to inner product space $\langle x, x \rangle$ is (1) $\lVert x \rVert = \langle x, x \rangle$ (2) $\lVert x \rVert^2 = \langle x, x \rangle$ (3) $\lVert x \rVert = \langle x, x \rangle^2$ (4) None of these
85.	Cayley-Hamilton theorem states that (1) Every square matrix satisfies its own characteristic equation (2) Every square matrix does not satisfy its own characteristic equation (3) Every rectangular matrix satisfies its own characteristic equation (4) None of these

Question No.	Questions
86.	If $\lvert z \rvert = \lvert z-1 \rvert$ then (1) Re (z) = 1 (2) Re (z) = 1/2 (3) Im (z) = 1 (4) Im (z) = 1/2
87.	The power series $\sum_{n=0}^{\infty} 3^{-n} (z-1)^{2n}$ converges if (1) $\lvert z \rvert \le 3$ (2) $\lvert z \rvert < \sqrt{3}$ (3) $\lvert z-1 \rvert < \sqrt{3}$ (4) $\lvert z-1 \rvert \le \sqrt{3}$
88.	An analytic function of a complex variable z = x + iy is expressed as $f(z) = u(x, y) + iv(x, y)$, where $i = \sqrt{-1}$. If $u(x, y) = 2xy$, then $v(x, y)$ must be (1) $x^2 + y^2$ + constant (2) $x^2 - y^2$ + constant (3) $-x^2 + y^2$ + constant (4) $-x^2 - y^2$ + constant
89.	$\int_{\lvert z \rvert=2} \frac{2z}{z^2+2} dz =$ (1) 0 (2) $-2\pi i$ (3) $4\pi i$ (4) 1
90.	The value of $\int_C \frac{\sin z}{4z+\pi} dz$ where C : $\lvert z \rvert = 1$ is a positively oriented contour. (1) 0 (2) $\frac{-\sqrt{2}\,\pi i}{4}$ (3) $\frac{-\sqrt{2}\, i}{4}$ (4) $\frac{-\pi i}{4}$

Question No.	Questions
91.	The non-empty set of real numbers which is bounded below has (1) supremum (2) infimum (3) upper bound (4) none of these
92.	The sequence $\{f_n\}$ where $f_n(x) = x^n$ is ________ convergent on [0, k], k < 1 (1) uniformly (2) pointwise (3) nowhere (4) none of these
93.	Every bounded sequence has at least one limit point. This represents (1) Archimedean Property (2) Heine-Borel theorem (3) Bolzano-Weierstress theorem (4) Denseness Property
94.	Which of the following is convergent ? (1) $\sum_{n=1}^{\infty} n^2 2^{-n}$ (2) $\sum_{n=1}^{\infty} n^{-2} 2^{n}$ (3) $\sum_{n=2}^{\infty} \frac{1}{n \log n}$ (4) $\sum_{n=1}^{\infty} \frac{1}{n \log(1+1/n)}$
95.	If a function f defined on [0, 1] as $f(x) = \begin{cases} i, \text{if } x \neq 1/2 \\ 0, \text{if } x = 1/2 \end{cases}$, then (1) f is not bounded (2) f is R-integrable (3) f is not R-integrable since f is not bounded (4) f is not R-integrable since lower and upper limits are unequal
96.	Let $f : R \to R$ be a monotone function. Then (1) f has no discontinuities (2) f has only finitely many discontinuities. (3) f can have at most countably many discontinuities (4) f can have uncountably many discontinuities

Question No.	Questions
97.	The length of an interval I is (1) Outer measure of an interval I (2) Less than outer measure of an interval I (3) Greater than outer measure of an interval I (4) Twice the outer measure of an interval I
98.	A set E is said to be Lebesgue measurable if for each set A (1) $m^*(A) = m^*(A \cap E) - m^*(A \cap E^c)$ (2) $m^*(A) = m^*(A \cap E^c) - m^*(A \cap E)$ (3) $m^*(A) = m^*(A \cup E) + m^*(A \cap E^c)$ (4) $m^*(A) = m^*(A \cup E) - m^*(A \cap E^c)$
99.	A non-negative measurable function f is integrable over the measurable set E if (1) $\int_E f = \infty$ (2) $\int_E f > \infty$ (3) $\int_E f < \infty$ (4) None of these
100.	If f is of bounded variation on [a, b] and $c \in (a, b)$. Then (1) f is of bounded variation on [a, c] and on [c, b] (2) f is not of bounded variation on [a, c] and on [c, b] (3) f is constant on [a, c] and on [c, b] (4) None of these

Mathematics Ph D Entrance Test Key

Sr. No	Set- A	Set-B	Set-C	Set-D
1	B	D	A	D
2	A	C	A	B
3	C	A	C	B
4	A	D	C	B
5	B	C	B	A
6	C	B	A	B
7	A	D	A	D
8	A	B	C	C
9	C	D	B	B
10	A	A	A	C
11	D	A	A	B
12	C	D	B	B
13	A	C	A	C
14	D	D	B	A
15	C	D	A	C
16	B	B	B	B
17	D	B	C	A
18	B	D	C	B
19	D	A	C	B
20	A	C	B	B
21	A	D	B	C
22	B	B	A	C
23	A	B	C	D
24	B	B	A	A
25	A	A	B	C
26	B	B	C	B
27	C	D	A	A
28	C	C	A	A
29	C	B	C	D
30	B	C	A	A
31	C	B	A	D
32	C	B	D	C
33	D	C	C	A
34	A	A	D	D
35	C	C	D	C
36	B	B	B	B
37	A	A	B	D
38	A	B	D	B
39	D	B	A	D
40	A	B	C	A
41	A	C	C	A
42	A	C	B	D
43	C	D	B	C
44	C	A	D	D
45	B	C	A	D
46	A	B	B	B
47	A	A	C	B
48	C	A	A	D
49	B	D	A	A

Sumeet 20/11/18 Anju 20/11/18 [illegible] Nov 20[illegible]

Poonam 20/11/18

50	A	A	C	C
51	B	A	C	C
52	B	B	C	B
53	C	A	D	B
54	A	B	A	D
55	C	A	C	A
56	B	B	B	B
57	A	C	A	C
58	B	C	A	A
59	B	C	D	A
60	B	B	A	C
61	C	A	D	A
62	B	A	B	D
63	B	C	B	A
64	D	C	B	A
65	A	B	A	C
66	B	A	B	A
67	C	A	D	A
68	A	C	C	C
69	A	B	B	C
70	C	A	C	A
71	D	C	A	A
72	B	B	D	A
73	B	B	A	C
74	B	D	A	C
75	A	A	C	B
76	B	B	A	A
77	D	C	A	A
78	C	A	C	C
79	B	A	C	B
80	C	C	A	A
81	A	B	D	A
82	D	A	C	B
83	A	C	A	A
84	A	A	D	B
85	C	B	C	A
86	A	C	B	B
87	A	A	D	C
88	C	A	B	C
89	C	C	D	C
90	A	A	A	B
91	A	A	B	B
92	D	D	B	A
93	C	A	C	C
94	D	A	A	A
95	D	C	C	B
96	B	A	B	C
97	B	A	A	A
98	D	C	B	A
99	A	C	B	C
100	C	A	B	A