Measure and Integration Theory

Max Marks: 20

Attempt any two questions.

I(a) Prove that Lebesgue measure is finitely additive.

(b) If E_1 and E_2 are measurable subsets of [a, b], then prove that

$m(E_1) + m(E_2) = m(E_1 \cup E_2) + m(E_1 \cap E_2)$

2(a) Show that a continuous function defined over a measurable set E is measurable. Is the converse of this theorem true?

(b) State and prove Lusin theorem.

3(a) Prove that if f and g are bounded and measurable functions on a set E of finite measure,

Then $\int_E f+g = \int_E f + \int_E g$

(b) State and prove Fatou's Lemma.

4(a) State and Prove Jordan Decomposition Theorem.

(b) Prove that if a function f is absolutely continuous in an interval [a, b] and if $f'(x) = 0$ a.e. in [a, b], then f is constant.

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Class: M.Sc. Mathematics 2nd Semester (DDE)   Subject: Operations Research Techniques
Paper code: 16MAT22C5   Marks: 20

Note: Attempt any two questions. All questions carry equal marks

Q.1 (a) What is Operation Research? Write a note on its origin and scope.

(b) In the context of Linear Programming Problem (LPP), define the followings:

(i) Basic Feasible Solution

(ii) Canonical Form of LPP

(iii) Surplus Variable

(iv) Dual of LPP

Q.2 (a) Solve the following LPP by using Simplex Method:

Max. $Z=3x_1 + 2x_2$

Subject to the constraints:

$x_1 + x_2 \leq 4$,

$x_1 - x_2 \leq 2$ and $x_1, x_2 \geq 0$

(b) Discuss the M/M/1 queueing model. Also, give its performances measures.

Q.3 (a) Find the basic feasible solution of the following problem using Vogel's Approximation method:

Origin/ Distribution Centre	1	2	3	4	5	6	Availability
1	4	6	9	2	7	8	10
2	3	5	4	8	10	0	12
3	2	6	9	8	4	13	4
4	4	4	5	9	3	6	18
5	9	8	7	3	2	14	20
Requirement	8	8	16	3	8	21	

(b) A company is producing a single product and selling it through five agencies situated in the different cities. All of a sudden, there is a demand for the product in five more cities that do not have any agency of the company. The company is faced with the problem of deciding on how to assign the existing agencies to dispatch the product to the additional cities in such a way that the travelling distance is minimized. The distances (in km) between the surplus and the deficit cities are given in the following distance matrix:

Surplus city \ Deficit city	I	II	III	IV	V
A	160	130	175	190	200
B	135	120	130	160	175
C	140	110	155	170	185
D	50	50	80	80	110
E	55	35	70	80	105

Determine the optimum assignment schedule.

Q.4(a) For what value of λ, the game with following payoff matrix is strictly determinable?

	B_1	B_2	B_3
A_1	λ	6	2
A_2	-1	λ	-7
A_3	-2	4	λ

(b) Determine the optimal strategies for each firm and value of the game.

Firm B

Firm A	B_1	B_2	B_3	B_4
A_1	35	65	25	5
A_2	30	20	15	0
A_3	40	50	0	10
A_4	55	60	10	15

Assignment

M.Sc. Mathematics-2nd

Paper Name:- Partial Differential Equations

Paper code:-20MAT22C4

Attempt any two questions **Total Marks 20**

1. Find the solution of the boundary value problem

$$\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial^2 x} \qquad (0 < x < L, t > 0)$$

$$u(0,t) = u(L,t) = 0$$

$$u(x,0) = \left\{ \begin{matrix} A(1-x); & 0 < x < \frac{L}{2} \\ 0; & \frac{L}{2} < x < L \end{matrix} \right\}$$

2. Find the solutions of non-homogeneous Poisson's equation

3. State and derive mean value formula for Heat equation.

4. Solve the boundary value problem

$6u_{x_1} + \ u_{x_2} = u^2$ in U where U is the real half space $(x_2 > 0)$ and $\Gamma = \{x_2 = 0\} = \partial U$

Assignment

M.Sc. Mathematics - 2nd

Integral Equations and Calculus of Variations

Attempt any two questions. **Total Marks: 20**

1. Find the eigen values and eigen functions of the integral equation

$$u(x) = \lambda \int_0^{2\pi} \sin(x+t)\, u(t)\, dt .$$

2. With the aid of resolvent kernel find the solution of the integral equation :

$$\phi(x) = x + \int_0^x (\xi - x)\, \phi(\xi)\, d\xi .$$

3. Explain the Construction of Green's function by variation of parameter method.

4. Find the extremal of the functional $J[y] = \int_1^2 \frac{\sqrt{1+y'^2}}{x}\, dx \quad y(1) = 0 , \; y(2) = 1.$

Assignment

M.Sc. Mathematics – 2^{nd}

Theory of Field Extension

Attempt any two questions. **Total Marks: 20**

1. If $a \in K$ is algebraic over F of odd degree show that $F(a) = F(a^2)$.

2. Determine the Galois group of the splitting field of x^4+1 over Q.

3. Prove that for every prime p and integer $n \geq 1$, there exists a field having p^n elements.

4. Let G be a group and H be a normal subgroup of G. Then if H and G/H both are solvable, then prove that G is also a solvable group.